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Chapter on LONG WAVES from book

IONOSPHERIC PROPAGATION OF RADIO WAVES

by

Ya. L. Al'pert

Institute of Terrestrial Magnetism, Ionosphere and
Propagation, U.S.S.R.

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U. S. DEPARTMENT OF COMMERCE
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A complete listing of the Bureau's publications can be found in National Bureau of Standards Circular 460, Publications of the National Bureau of Standards, 1901 to June 1947 ($1.25), and the Supplement to National Bureau of Standards Circular 460, July 1947 to June 1957 ($1.50), and Miscellaneous Publication 240, July 1957 to June 1960 (Includes Titles of Papers Published in Outside Journals 1950 to 1959)($2.25); available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C.

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Chapter on <u>LONG WAVES</u> from book

IONOSPHERIC PROPAGATION OF RADIO WAVES

by

Ya. L. Al'pert

Institute of Terrestrial Magnetism, Ionosphere and
Propagation, U.S.S.R.

Academy of Science, U.S.S.R.
(1960)

Translation by
The late Dr. Alex Nennsberg

Translation Editors
J. B. Reubens and J. R. Wait

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NATIONAL BUREAU OF STANDARDS
BOULDER LABORATORIES
Boulder, Colorado

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Preface

This translation of Chapter 7 of Ya. L. Al'pert's book
"Rasprostvanenie Radiovoln i Ionosfera," was supported by the
Advanced Research Projects Agency. This chapter deals primarily
with the propagation of very low frequency radio waves. Also in-
cluded are Appendices 6 and 7 of the book since they also deal with
long waves. Just before completing these translations Dr. Alex
Nennsberg passed away. Mr. J. B. Reubens and I have attempted to
carry out the editing of Dr. Nennsberg's literal translation. At the
same time, I have taken the liberty to add a number of editorial foot-
notes which might be of interest to the reader. Mrs. Eileen Brackett
has provided valuable help in the typing and preparation of this report.

James R. Wait
August, 1961

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Long Waves

As previously pointed out, long radio waves cover the range between 2,000 to 20,000 meters. ~~However,~~ the fundamental peculiarities of long waves, which separate them from medium waves, appear in practice, starting with waves between 5 and 6 km in length and greater, at relatively large distances from the source.

The height at which the ionosphere begins at these low frequencies is composed of only several wavelengths. The transition region at the base of the ionosphere, where electron density and conductivity increase rapidly, is of the order of one wavelength or much smaller. For this reason, the D-region of the ionosphere is not an absorbing layer for long waves, as it is for the rest of the waves in the radio frequency range, but serves as a conductive wall of the waveguide, directing the flow of radiation, and, by this effect, contributing to long distance propagation. Long waves in the earth-ionosphere waveguide are excited as a spectrum of waves. Even near the radiator, the interfering character of the field structure, depending on distance, caused by the superposition of these waves, may be observed. At low radio frequencies, at a point sufficiently remote from the source, direct and reflected waves from the ionosphere seldom appear separately. For example, at distances above 2000-3000 km, only one sufficiently intense wave remains. However, this so-called zero mode of the spectrum of excited waves is of a purely waveguiding nature and is not a direct wave. Hence, the classical approach, based on the strict solution of the diffractional problem (see para. 22), as well as ray-tracing interpretation of the picture of propagation, taking into account the influence of the ionosphere, are often not suited for describing the propagation of long waves.

In recent years, the theoretical and experimental investigation of long waves has been considerably broadened. A special interest has been shown in the propagation of electromagnetic waves of low and super-low frequency. In particular, such problems arise in the study of the so-called atmospherics: solitary signals, excited by lightning discharges propagating along the earth, and also of other signals of similar type. These considerations are also important in the study of the so-called "whistling" atmospherics in the same frequency range. These are excited by lightning discharges which leak through the whole thickness of the lower part of the ionosphere and then propagate along the force lines of the magnetic field of the earth in outer space, traveling many thousands of kilometers before

their return to the earth.

After an interval of almost 40 years, interest in the study of long waves has revived. It is precisely during this latter period, however, that a physical theory of propagation of long waves ~~has~~ was ~~been~~ developed which provides methods of computation and makes it possible to compare theory with experiment. ~~At the same time,~~ Also new experimental results ~~have been~~ obtained from ~~two~~ sources: direct experiments with radio waves, and indirectly by using atmospherics. Therefore, at present, it is not necessary to be limited to radio waves created by radio arrangements only, but it becomes possible to broaden the investigation down to electromagnetic waves at the lowest observed frequency (i.e., 10-12 kc/s).

~~In this connection,~~ presently some results of theoretical computations are ~~shown below,~~ including super-low frequencies down to some tens of cycles per second. The results of experimental investigations of atmospherics, particularly the whistling atmospherics, ~~which are of interest within the scope of this book,~~ are briefly analyzed. It should be noted that the question of electromagnetic waves, in the range 3-5 kc/s and lower, remains uninvestigated with adequate precision, either theoretically or experimentally up to the present time. (In connection with this, see Supplement 7.)

29. Some Results of Experimental Investigations

The properties of medium/long waves differ in many aspects from the properties of waves 10-25 km long and longer. For example, at wavelengths from 2-3 km long and somewhat longer, the direct wave, almost in pure form, prevails considerably up to distances between 1000 to 2000 km in daytime. After sunset, the intensity of the field increases strongly and other specific peculiarities of medium waves appear (see para. 33). At the same time, the expected normal diurnal variation of average values of the field strength of waves between 15-20 km in length is frequently absent. Therefore, in the following discussion of fundamental peculiarities of long radio waves, their differences at the various frequencies in this range should be kept in mind.

1. Dependence of Tension of the Field on Distance

Even at short distances from the radiator, both in the daytime and at night, the interfering character of the field, depending upon

distance, has an effect on long waves.

Figure 233 shows the results of measurements at higher frequencies; here, at a frequency of 85 kc/s ($\lambda \approx 3500$ m). The quasiperiodical oscillations of the field become observable at distances from the radiator greater than 100 wavelengths, and increase with distance. These variations of waves can be interpreted as a result of the superposition of the direct and once-reflected wave from the ionosphere. Indeed, the coefficient of reflection increases with the angle of incidence of a wave on the ionosphere, which leads to strengthening of the influence of the reflected wave. Moreover, analysis of the results of such measurements shows that the oscillations of the field frequently occur close to the curve of the ground wave. At low and super-low frequencies, the oscillations of the field are greater and become already observable at a distance of several wavelengths from the radiator. Thus in measurements made at a frequency of 16 kc/s ($\lambda \approx 18,700$ km) (Fig. 234), the amplitude of the field changed by 25 percent even at a 5-6 λ distance from the radiator; in the preceding figure, oscillations of approximately 10 percent began to be noticeable only at a distance where $r \approx 70 \lambda$, and oscillations of 25 percent only at the point where $r \approx 110 \lambda$. At distances between 100-200 km and more, at a frequency of 16 kc/s, it was impossible to interpret the results of the measurements exactly by introducing the direct and reflected waves into the analysis.
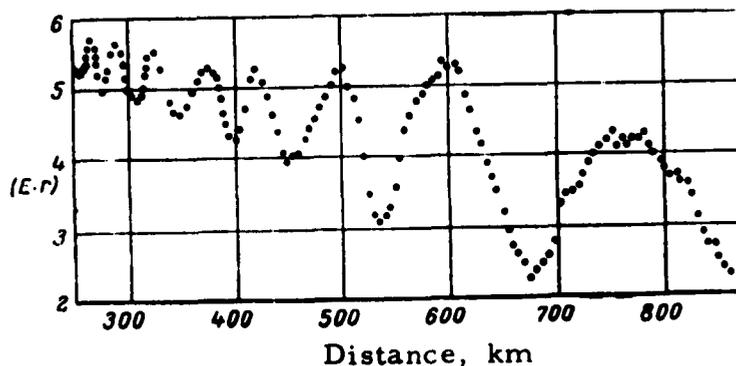
Fig. 233 - Product of field strength with distance from radiator. (Measurement on frequency 85 kc/s ($\lambda \approx 3500$ m)

Translator's Note: Henceforward, many figures and diagrams in the text are supplied with Russian inscriptions along the curves and margins. Not being able to reproduce the figures in full, this translator puts them in English in the same order as they appear on the original, from left to right. The explanatory legend will be translated after the number of the figure.
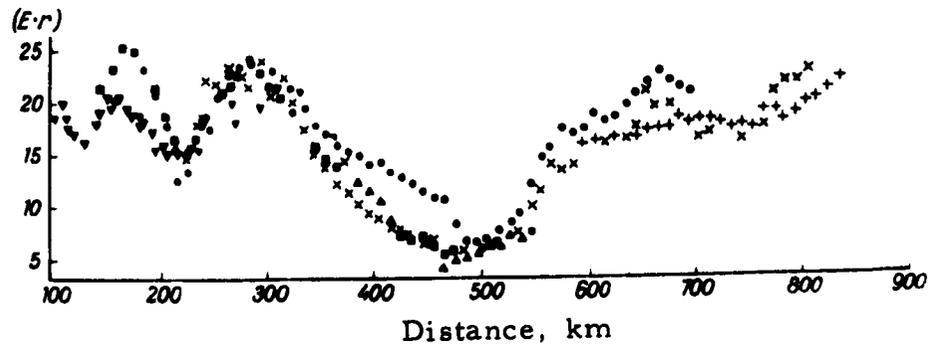
Fig. 234 - Dependence of (E · r) with distance. Measurement on frequency 16 kc/s ($\lambda \approx 18700$ m)

With increased distance, the variation in field strength gradually smoothes and becomes approximately equal at various frequencies, the amplitude of the field being somewhat larger at the lower frequencies. Experimental averaged curves of field strength, depending on distance (Figs. 235 and 236), were obtained from the data of many measurements [160] conducted at different periods of time over the sea, up to a distance of 12,000 km. In Fig. 235, the average values of the field strength obtained for day and mixed routes, at a frequency of 18 kc/s, are compared with the formula of ideal radio transmission, $300\sqrt{W}/r$, and with the curve, E(r), computed for the spherical earth. It is evident that as distance increases, the experimental values of E exceed the values of E in the direct wave by two to three magnitudes or more.

The same discrepancy is observed for routes situated completely in the region of darkness; the corresponding results of measurements (Fig. 236) are averaged over the range, 12.5-30 kc/s. From Fig. 235, it is evident that the amplitude of the field may change several times in various cases, and that the changes do not depend on frequency.

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Kitchen, F.A., B.G. Pressey, and K.W. Tremellen, A review of present knowledge of the ionospheric propagation of very low, low-and medium waves, J. Inst. Elec. Eng., Pt. III, 100 (1953).
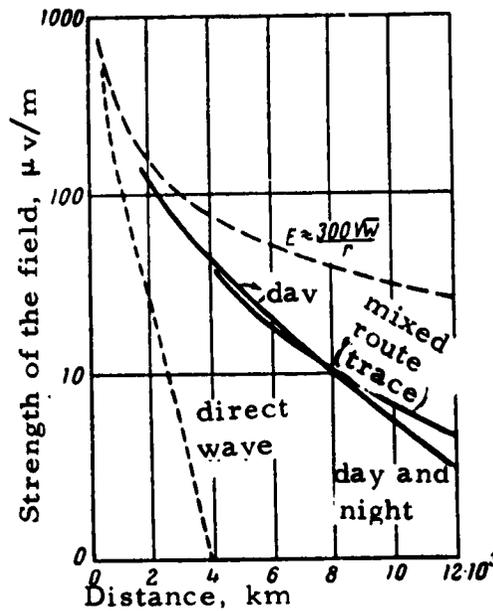
Fig. 235 - Averaged results of measurements of field strength showing dependence with distance on frequency 18 kc/s (solid line). By dotted line is shown the course of theoretical curves. (W - 1 kw)
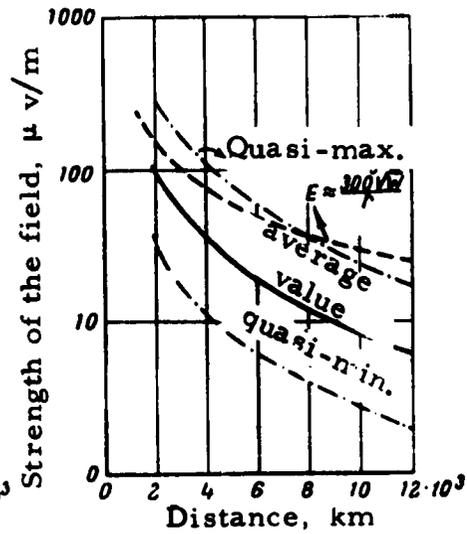
Fig 236 - Averaged results of measurements of field strength showing dependence with distance and range (diapasone) 12. 5 - 30 kc/s (solid line).

Until recently, various types of empirical formulae, obtained as the result of processing a series of numerous measurements, were still used frequently in the computation of tension of the field. One such is the formula of Austin-Cohen

$$E = \frac{300 \sqrt{W}}{r} \sqrt{\frac{\theta}{\sin \theta}}\ e^{-\frac{0.0015}{\sqrt{\lambda}} r} \qquad (29.1)$$

or Espenschied, Anderson and Bailey

$$E = \frac{300 \sqrt{W}}{r} \sqrt{\frac{\theta}{\sin \theta}}\ e^{-\frac{0.005}{\lambda\,1.25} r}. \qquad (29.2)$$

There are supposed to be present in the exponents of the formulae of type (29.1), multipliers $(- 0.0014 \, r/\lambda^{0.6})$, or $(0.0045 \, r/\lambda^{1.4})$, which were selected each time to satisfy the series of experiments under scrutiny in the paper.

Other types of empirical formulae were also used whose derivation is based on qualitatively correct perception of the formation of cylindrical waves, directed by the earth and ionosphere, in the earth-ionosphere waveguide. Starting from this point, and taking into account the sphericity of the earth, the flow of energy from the sources crosses the lateral surface

$$S_o = \pi \sin \theta \, (2 R_o z + z^2) \tag{29.3}$$

(Fig. 237), differing from the lateral surface, $S = 2\pi \, r \, z$, in the case of a flat earth. Not taking into account losses in the walls of the waveguide, we have

$$\frac{C}{4\pi} \, E^2 = \frac{W}{S} , \tag{29.4}$$

and for the vertical dipole (its intensification is equal 3/2)

$$E = \frac{300 \sqrt{W}}{\sqrt{z} \sqrt{r}} \, \sqrt{\frac{\theta}{\sin \theta}} \quad \text{mv/m,} \tag{29.5}$$

where $r = R_o \theta$, and z are expressed in km; W in kw; and $\dfrac{z}{R_o} << 1$.
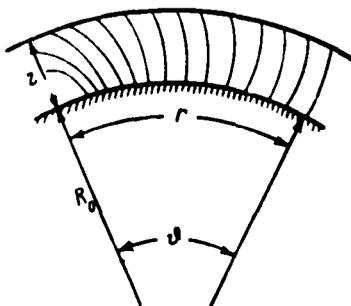
Fig. 237 - Illustrating the deduction of the
formula taking into considera-
tion the sphericity of the earth.

In order to account for the losses in formula (29. 5), various authors have used exponential multipliers of the type, $\exp(-a \, r/\lambda \, m)$ and have selected values of a and m which would best satisfy the results of the experiments.

Naturally, at present, there is no need to use formulae of this type since strict theoretical calculations result in formulae which permit one to compute not only the field strength of long and super-long waves, but also their phase structure for a wide range of frequencies, and their dependence on the conductivity of the ionosphere and the earth (see para. 30).

2.   Various Properties of Long Waves

As we have seen, the ionosphere contributes to a considerable increase of intensity of long radio waves over great distances, and to the interference characteristics of the field.   Let us investigate the other properties of long radio waves, also conditioned by the influence of the ionosphere, mainly by the changeability of its state.

At distances near the radiator, where the influence of the direct wave is still felt, even at the lowest frequencies of this range, it is possible to consider the field as a superposition of direct and reflected waves from the ionosphere.   Thus, by compensating for the direct wave in the experiments, the ionospherically reflected wave may be separated.   This has permitted the determination in various experiments of the state of polarization of long waves, their apparent height, and their coefficient of reflection at vertical and oblique incidence [161].

Summarizing, it was shown by various methods, (the experiments were conducted within the range $\lambda \approx$ 2-18, 7 km), that up to distances between 100-300 km at middle latitudes, the reflected wave is almost circularly polarized.   At greater distances, the state of polarization is changed.   For instance, at $\lambda \approx$ 18 (7 km), and $r \approx$ 400 km, purely linear polarized waves are already observed.   In this case, the plane

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   Bracewell, R. C., K. G. Budden, J. A. Ratcliffe, T. W. Straker and
   K. Weekes, The ionospheric propagation of low- and very-low fre-
   quency radio waves over distances less than 1000 km, J. Inst. Elec.
   Eng., Pt. III, 98, 221 (1951).

of polarization has turned approximately 45° relative to the plane of incidence of the wave.

The coefficient of reflection, ρ, of these waves also changes considerably with distance. Figure 238 shows the experimental dependence of the coefficient of reflection on frequency, obtained under various conditions, almost at vertical incidence; Figs. 239 and 240 give the idea of its diurnal and seasonal changeability at various frequencies. With each increase in distance, the coefficient of reflection increases strongly. For instance, 16 kc/s at r $\approx$ 300 km in the described experiments gave a measured value of $\rho \approx 0.15$, and at the distances, r $\approx$ 500-800 km, a value of $\rho \approx 0.33$. In this experiment, the coefficient of reflection was normally observed to increase suddenly at a distance of about 400 km. In summer, at frequencies of 85 and 127 km kc/s, the value of $\rho \approx 0.001$ at vertical incidence; when r $\approx$ 400 km, $\rho \approx 0.01$, when r $\approx$ 700 km, $\rho \approx 0.04$, and when r $\approx$ 900 km, $\rho \approx 0.055$. In line with this, a rapid decrease of ρ is observed, over long distances, one hour before sunrise, and a rapid growth of ρ in the period of sunset. The apparent altitude of reflection with this range of frequencies changes within limits between 67-85 km. At the lower frequencies, altitude increases from 10-16 km when passing from day to night. These data result from observations at close and vertical incidence of the wave. At oblique incidence, at 16 kc/s, the same diurnal variations of altitude are observed. However, at frequencies of 100 kc/s and more, the diurnal changes in apparent altitude, instead of being on the order of 7-10 km, are about 18 km. The character of modification of all the given values does not lend itself to a simple explanation. It suggests that the lower part of the ionosphere (its base), while playing a big part in the propagation of long radio waves, has a series of peculiarities which have been little studied up to now. This, in great part, is because up to now there were no direct measurements made of distribution of charges and number of collisions at these altitudes.

Now, let us pass to the study of other properties of long radio waves. The state of the field of long waves is subject to chaotic variations (so-called fadings or black-outs), as well as to regular changes (diurnal, seasonal, and eleven-year cycle). Definite effects are also observed, conditioned by variations of the magnetic field of the earth, sudden atmospheric disturbances, etc. All of these effects are studied most completely as a part of measurements of the tension of the field.
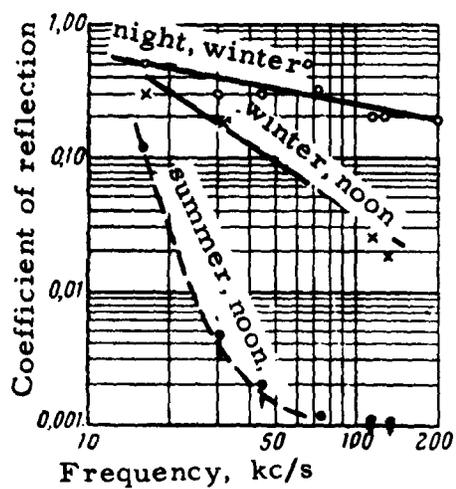
Fig. 238 - Dependence of coefficient
of reflection with frequency.
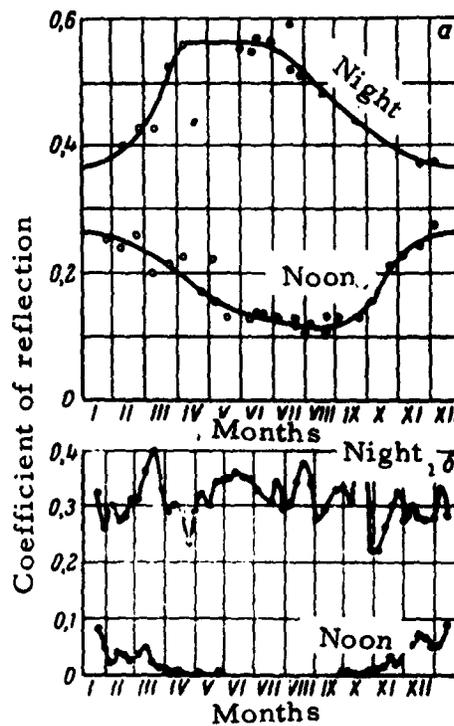
Fig. 239 - Seasonal course of
coefficient of reflection
a - 16 kc/s; b - 70 kc/s
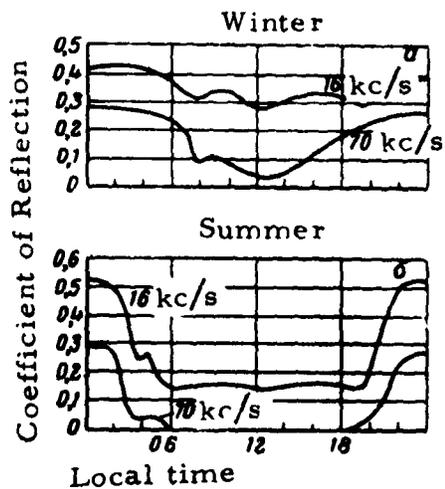
Fig. 240 - Diurnal course of coefficient
of reflection

Oscillations in amplitude which are relatively slow and shallow gradually increase with the shortening of wavelength and become very noticeable, especially in the range of medium waves. Similar oscillations are observed in the recording of phase and direction of arrival of radio bearing waves (see, for instance, Fig. 241).
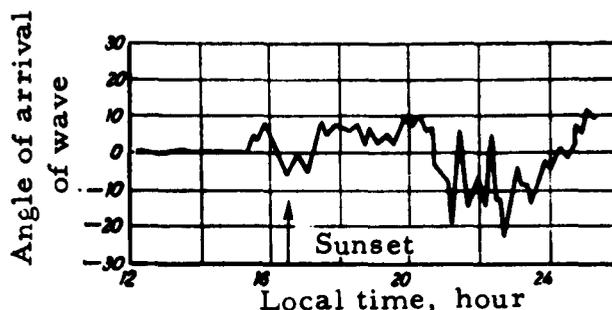
Fig. 241 - Oscillations of radio bearing on wave 4700 m on the distance 837 km from radiator.

Let us note here, that according to a series of data for distances between 5000-6000 km, the phase of the received wave is almost constant. This, seemingly, points to the absence at this range of any influence by many modes.

In many cases, however, the diurnal variation in amplitude is subject to many changes. Figure 242 shows, for comparison, the curves of diurnal variation of average monthly values of field tension, E, for three stations 20 kwt strong (λ = 5270, 11,680, and 17,500 m) over a distance of 5482 km.

From this figure (drawing), it is evident that with the increase in wavelength, the limits of change of E are decreased, and that at all wavelengths, the value of E increases at nighttime. However, it is known that over some traverses a decrease of E at nighttime is regularly observed, not an increase. The characteristic peculiarity of the diurnal variation in the field amplitude of long waves is, above all, the considerable decrease in E during the period of sunrise. This phenomenon is established over a wide range of wavelengths, beginning as low as those at several hundred kilometers.
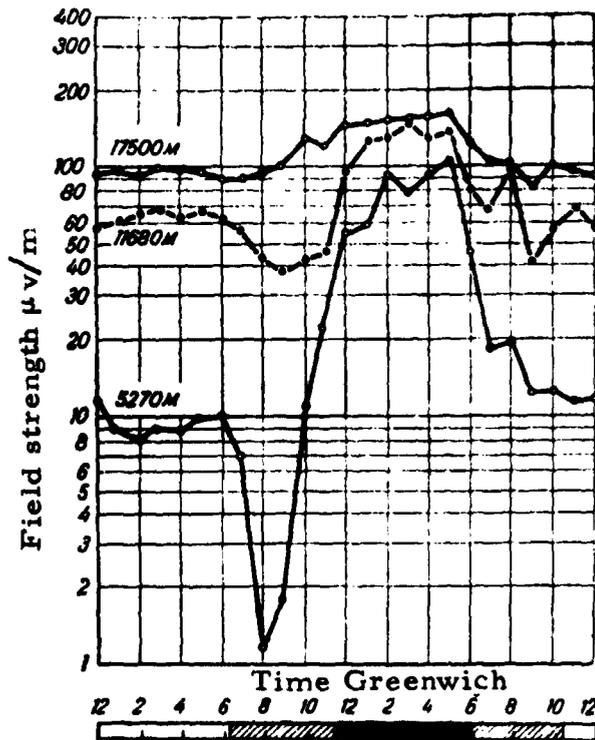
Fig. 242 - Average monthly diurnal curves of field strength on three waves at distance 5482 km

In a number of cases, at the moment of sunrise, the amplitude of the field has a deep minimum with following maximum. After this, a relatively constant value of E is established, which, in its dependence on distance, or on wavelength, may be large or smaller than the night value of field amplitude. A similar phenomenon is described in the literature as "twilight effect", because it is frequently observed during sunset. As a rule, it is less well-marked at this time. Twilight effect is absent altogether, both in the morning and evening, on many strip-chart recordings (Fig. 243). The seasonal course of field amplitude is also subject to changes. Amplitude of field is greater in daytime over many paths, and greater in summer than in winter; at nighttime, it is approximately the same all year round.
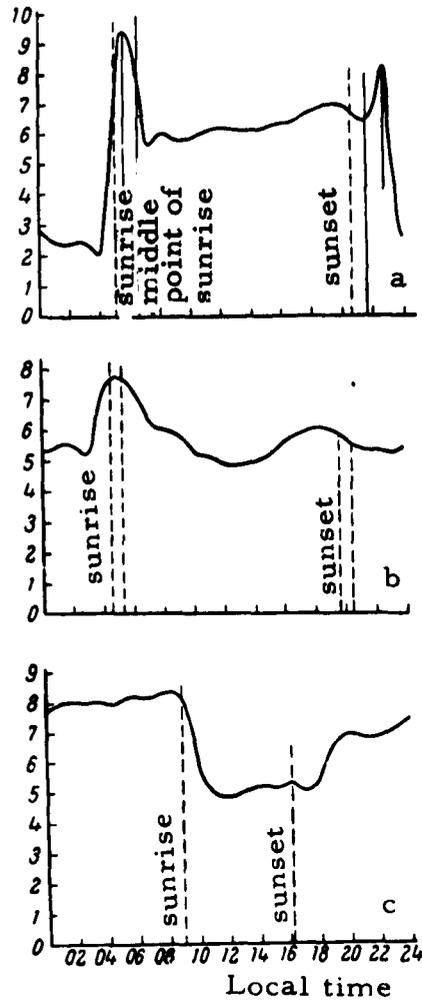
Fig. 243 - Diurnal course of field amplitude
a - on frequency 16 kc/s on distance 2500 km;
b - on distance 17. 8 kc/s on distance 7860 km;
c - on frequency 26 kc/s on distance 11200 km.

There are some indications that the amplitude of field is greater over paths situated at the same latitudes than over paths along the same meridians.

All these effects are explained by the complicated structure of the field of long radio waves, depending upon distance from the radiator, degree of illumination of route, height of the base of the ionosphere, character of transition of altitude into region of semi-darkness, and similar phenomena.

The periods of sudden disturbances in the ionosphere are characterized by the fact that at this time there occurs a pronounced weakening of the field of short waves; the field strength of the long waves, on the contrary,often increases exactly at this time. This can be seen in Fig. 244, which shows an increase of the field amplitude at a wavelength of 11,000 meters during a period of sudden disturbance which began at 11 h 57 m. At the time of sudden disturbances, as we have seen, the structure of the E- and R-regions almost does not change; only their electronic and ionic concentrations increase, with a consequent increase in the gradient of ionization of the D-region. Therefore, at this time, the absorption of waves passing through the D-layer increases, and the losses in intensity of long waves directed by this layer may decrease because they leak into the D-layer less. With an increase of frequency in this range, a decrease , and not an increase, in the amplitude of the field may well prevail at the time of sudden disturbances.
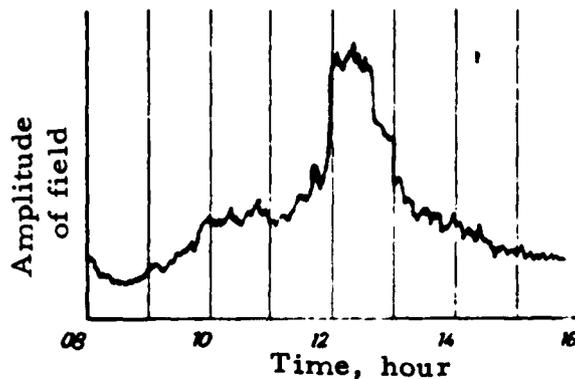
Fig. 244 - The growth of the amplitude of field on wave 11100 m in the period of sudden disturbance which began at 11 h 57 m.

Numerous observations have shown that with the increase in activity of the sun and magnetic storms, the field strength of the long waves (Fig. 245) increases; the reverse is observed for medium waves. Such dependence is also apparently explained by the increase in the gradient of electronic concentration in the base of the ionosphere during these periods.
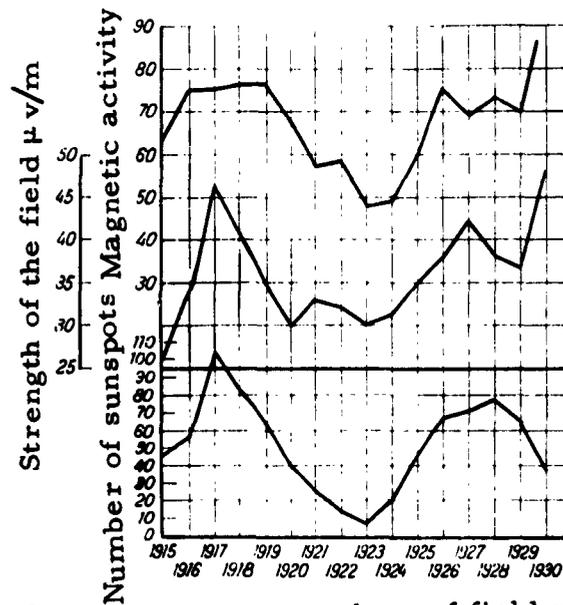
Fig. 245 - Course of average values of field strength in the range
10000 - 20000 m, number of sunspot and intensity of field for a
solar cycle.

30. Theoretical Computations of the Field

In general, computations of the field of long radio waves de-
mand a solution of the waveguiding problem for a spherical earth,
surrounded by a nonhomogeneous ionosphere, the properties of
which are modified substantially with frequency.   In such a situation,
it is difficult to solve this problem mathematically.   Up to now, no
one had succeeded in reducing the problem to simple form which
would permit a discussion of the experimental data on this basis.
At the same time, a great number of papers were devoted to this
problem [162 - 164].   In the latest series of investigations

162

Watson, G. N. , The transmission of electric waves around the
earth, Proc. Roy. Soc. , 95, 546 (1919).

163

Schumann, W. , Uber die Oberfelder bei der Ausbreitung langer
elektrische Wellen in System Erde-Luft-Ionosphare, Zs. f. angew.
Phys. , 4, 474 (1952); 6, 35,225, 267, and 346 (1954).

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Krasnushkin, P. E. , N. A. Yablochkin, The theory of propagation of
very long waves applied to the problem of long distance communi-
cation and navigation, Proc. of the State Union, Scientific Research
Institute (1955).

[163, 165], such a spherical problem was formulated for a homogeneous ionosphere. In the process of computation, however, the authors preserve the influence of the sphericity of the earth only by $\theta/\sin\theta$. Thus, as special cases, they obtain previously investigated solutions for the plane problem with a uniform upper boundary [166, 167]. In these papers, allowing for sphericity is reduced to taking into account a "geometric" factor, also obtainable from simple considerations (see deduction (29. 5)).

Consequently, the spherical problem for a homogeneous ionosphere is solved in one of the most recent papers [169] for sufficiently high conductivity of earth when the approximate boundary conditions are suitable. The equation obtained in this paper, defining the wave numbers of wave spectrum, is rather complicated. The author does not solve it. However, using this equation approximately, it is possible to correct the solution for the flat earth and, consequently, take into account the influence of its sphericity. (See Supplement 6 in which results of this work, and also of paper [242], published after this present work went to print, are given briefly.

163
  Schumann, W. , Uber die Oberfelder bei der Ausbreitung langer elektrische Wellen in System Erde-Luft-Ionosphare.
165
  Wait, J.R. , The mode theory of VLF ionospheric propagation for finite ground conductivity, Proc. IRE, 45, 760-768 (1957).
166
  Brekhovskikh, L. M. , Waves in stratified media, Acad. Sci. , SSR, 1957; Bul. Acad. Sci. , Physics Series, 8, (1949, p. 505; 13, pp. 515-534.

167
  Budden, K. G. , The propagation of a radio atmospheric, Phil. Mag. , 42, 1 (1951); 43, 1179 (1952).
169
  Bremmer, H. , National Bureau of Standards Report No. 5518 (1957).
242
  Wait, J.R. , Terrestrial propagation of very low-frequency radio waves, a theoretical investigation, J. Research NBS 64D (Radio Prop.), No. 2, 153-204 (Mar. -Apr. 1960).

One of the basic conditions, without which it is impossible to solve the waveguiding problem for long radio waves correctly, is the need to take into account the nonhomogeneity and dispersion of the ionosphere, and the dependence of its reflective properties on frequency which is modified considerably in the frequency range under consideration. It is very difficult to solve such a problem mathematically in actual form; all the more since the analytical formulae describing the modification of corresponding parameters of the ionosphere are not known. Therefore, one is forced to use methods of computation based on the physical picture of phenomena, and to describe the real properties of the ionosphere with the aid of effective parameters and equivalent parameters of a homogeneous ionosphere. Such a method of computation for a flat earth is laid down in paper [168]. As analysis has shown, it can be reconciled very well with experimental data over a wide range of frequencies. The method of computation is given briefly below, together with results of some comparisons between numerical computations and experimental data.

1. Method of Computation

The method for nonhomogeneous plane waves, used previously for computing the field of the dipole over the flat earth [170], is added to the basic steps for solution of the flat waveguiding problem. This method has an obvious physical basis. Let us describe it briefly, since this permits us to understand how the solution for the homogeneous, sharply bounded ionosphere is transferred directly to the case of a nonhomogeneous (stratified) ionosphere.

As is known, the field of an elementary dipole, proportional in free space to $\exp(ikR)/R$, can be resolved into a continuum of non-homogeneous plane waves. Therefore, formally, the field of a dipole, situated over or near the boundary of separation, may be described as a result of superposition of a direct wave and of a continuum of non-homogeneo s plane waves reflected from this boundary. The coefficient of reflection from the separation boundary is generalized in

168

Al'pert, Ya. L. , On the propagation of low frequency electromagnetic waves over the earth's surface, Bul. Acad. Sci. , USSR, Phy. Series, 1955; Progress in Physics, 60 (1956), p. 369.

170

Weyl, H. , Ausbreitung elektromagnetischer Wellen uber einen ebenen Leiter, Ann. d. Phys. , 60, 481 (1919).

this case in such a way that the sines and cosines of angle of incidence are substituted for by their complex values. These, in turn, serve as arguments of plane nonhomogeneous waves. The corresponding consideration, applied to a plane waveguide, leads to the appearance of four groups of plane nonhomogeneous waves [166 - 168] whose superposition gives the following values of field when the source and the point of observation are located on the boundary of the separation: Vertical and horizontal components of electric and magnetic fields

$$E_z = i \frac{120 \pi^2 Ih}{\lambda z} \sum_{n=0}^{\infty} S_n^3 \, H_0^{(2)} (k \, S_n \, r) \, P (C_n),$$

$$(30.1)$$

$$H_\varphi = \frac{120 \pi^2 Ih}{\lambda z} \sum_{n=0}^{\infty} S_n \, H_1^{(2)} (k_o \, S_n \, r) \, P (C_n),$$

where E_z and H_φ are expressed in mv/m; where h is the effective height (of the transmitting antenna), z is the height of the ionosphere; λ is the wavelength; r is the distance in km; and I is the current in amperes. In formula (30.1), $H_0^{(2)}$ and $H_1^{(2)}$ are Hankel's functions with the function, $P(C_n)$ given by

$$P(C_n) = \frac{1}{2} \frac{[1 + \rho_1 (C_n)]^3 \, \{\rho_1 (C_n)\}^{-1}}{\left\{1 + \frac{i}{2h \, k_o \, \rho_1} \frac{\partial \rho_1}{\partial C} + \frac{i}{2n \, k_o \, \rho_2} \frac{\partial \rho_2}{\partial C}\right\} C_n} \qquad (30.2)$$

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Brekhovskikh, L. M. , Waves in stratified media.

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Budden, K. G. , The propagation of a radio atmospheric.
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Al' pert, Ya. L. , On the propagation of low frequency electromagnetic waves over the earth's surface.

$\rho_1(C)$ and $\rho_2(C)$ are formulae of coefficient of reflection from the surface of the earth and the ionosphere.[†] The complex numbers, (S_n and C_n), expressed as

$$S_n = \sqrt{1 - C_n^2} = S_{n_1} + S_{n_2} \quad , \qquad (30.3)$$

are sine and cosine, and define the wave numbers, $k_o S_n$ of the spectrum of cylindrical waves, excited in the earth-ionosphere waveguide. They are computed from the equation of poles, obtained from the solution of this problem and having the form

$$2h\, k_o\, C_n + i \ln [\rho_1(C_n)\, \rho_2(C_n)] = 2n\,\pi \quad . \qquad (30.4)$$

We now use the asymptotic forms of the Hankel functions, which are well satisfied, if

$$|k_o S_n r| > 4 \sim 5, \quad 2\pi < \arg(k_o S_n r) < \pi \quad , \qquad (30.5)$$

then introducing the multiplier $300\,\sqrt{W}$ instead of $120\,\pi I\, h$, and also the geometric factor, which takes into consideration the sphericity of the earth, we obtain

$$E_z = \frac{300\,\sqrt{W}\,\sqrt{\lambda}}{z\sqrt{r}}\,\sqrt{\frac{\theta}{\sin\theta}}\,\sum_{n=0}^{\infty} S_n^{3/2}\, P(C_n)\, e^{-i\left(k_o S_n r - \frac{3\pi}{4}\right)} \quad ,$$

$$\qquad (30.6)$$

$$H_\varphi = \frac{300\,\sqrt{W}\,\sqrt{\lambda}}{z\,\sqrt{r}}\,\sqrt{\frac{\theta}{\sin\theta}}\,\sum_{n=0}^{\infty} \sqrt{S}_n\, P(C_n)\, e^{-i\left(k_o S_n r - \frac{3\pi}{4}\right)} \quad .$$

[†] The factor $1/2$ in eqn. (30.2) should be $1/4$. It should be noted that this factor was only given in the author's previous work [168] for $\rho_1 = 1$. Its generalization appears in reference [242], (JRW).

Thus, it is evident that the field is a superposition of the spectrum of cylindrical waves, decreasing in a proportion of the form

$$\frac{1}{\sqrt{r}} \; e^{-k_o S_{n_2} r} \quad .$$

In formula (30.6) the function standing under the sum is an analogue of the attenuation function $f(\rho)$, but in contrast to $f(\rho)$, its modulus does not decrease monotonically with distance, but oscillates. Therefore, the formula

$$B(\omega, r) \; e^{-i\Phi(\omega, r)} \approx \sum_n \sqrt{S_n} \; e^{-i\left(k_o S_n r - \frac{3\pi}{4}\right)} \qquad (30.7)$$

may be called an interference multiplier of the field of radio waves in the waveguide.[†]

The formulae given above show that, for the computation of field of long waves, it is necessary to determine the wave numbers $k_o S_n$. These are obtained by solving a rather complicated transcendental equation of the poles (30.4). Here $\rho_1(C)$ is the Fresnel coefficient from the earth. As for the form of function, $\rho_2(C)$, it is unknown, inasmuch as the coefficient of reflection from the nonhomogeneous ionosphere, generally speaking, modifies the various boundary conditions. Having this difficulty in view, it is possible to solve this problem by the following: The equation (30.4) is first computed for a sufficient range of electric parameters of the ionosphere and various frequencies, selecting as $\rho_2(C)$ the Fresnel coefficient of reflection from the homogeneous medium. This permits us to obtain the dependence of poles, C_n, on these parameters. Then we compute for various frequencies, and for a given real model of ionosphere, the true values of modulus and of the argument of reflection coefficient. Next, after comparison with the corresponding values of Fresnel's coefficient of reflection, we compute the dependence of the medium

[†]

Apparently in writing eqn. (30.7), $P(C_n)$ has been set equal to one. (JRW)

on frequency and electric parameters. Thus, the effective values of corresponding parameters give, with the help of Fresnel's formula, the same values of intensity and phase for the reflected wave. The effective value of the coefficient of refraction found by this method is used later on for determination of S_n and C_n. This method was followed in paper [168].

Below are the corresponding results of numerical computations for vertically polarized waves. The generalized Fresnel formula has the form:

$$\rho_2(C) = \frac{n_2^2 C - \sqrt{n_2^2 - 1 + C^2}}{n_2^2 C + \sqrt{n_2^2 - 1 + C^2}} \qquad (30.8)$$

neglecting the influence of the magnetic field of the earth. Then, if $\omega^2 << \nu^2$,

$$n_2^2 \approx \left(1 - \frac{4\pi N e^2}{m \nu^2}\right) - i\, \frac{4\pi e^2}{m \omega}\, \frac{N}{\nu} . \qquad (30.9)$$

Since the first term in (30.9) is near to one, then for the lower part of the D-region, which plays the basic part in transmission of long radio waves in daytime, it is suitable for this time of day to select relation $(N/\nu)_{eff}$ as the effective parameter of medium [168], defined by adjusting Fresnel's coefficient to the true coefficient of reflection. Therefore, if the theoretical dependences of $\rho_2(C)$ and C_n on (N/ν) are established, then the first of them determines (N/ν_{eff}), and the second, the corresponding value of C_n to it.

2. Dependence of Field Strength Distance and Frequency

We have seen that the structure of the field of long waves can be described by interference multiplier (30.7). The analysis of the modulus (30.7), shows that the amplitude of field, $B(\omega, r)$, changes in an irregular way, depending upon distance from its source, since the spatial periods, or the lengths of waves, λ/S_n, of various components of the spectrum are comparable. On moving away from the

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source, the factor $B(\omega, r)$ gradually becomes quasi-periodic since the majority of spectrum waves (modes) are damped with the exception of the first two or three (Fig. 246). The amplitude of field changes in a number of details with relatively small changes in ionospheric parameters. This is especially true near the radiator (Fig. 246 a, b, c.). The same phenomenon is observed with small changes of frequency in constant ionospheric parameters (Fig. 247).
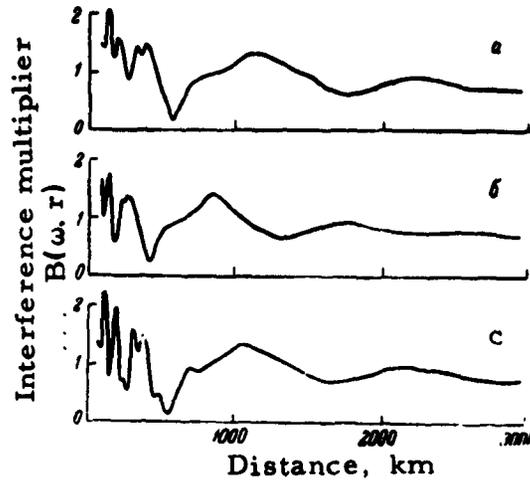

Fig. 246 - Dependence of modulus of interference multiplier $B(\omega, r)$ from distance for various values of N/v on frequency of 15 kc/s.
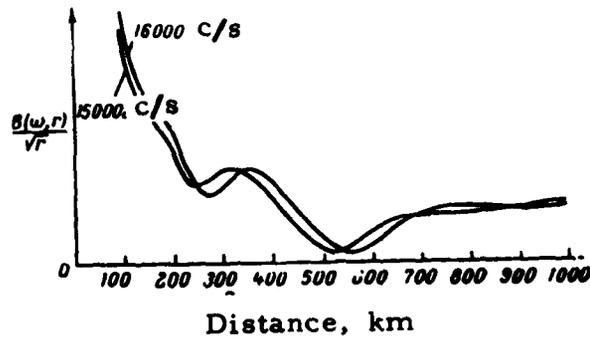

Fig. 247 - Dependence $B(\omega, r)\sqrt{r}$ on distance on two near frequencies.

The results of computations, conducted for real modes of the ionosphere, showed that, in daytime, beginning with 30 kc/s and below, and over distances exceeding 1000 km, the basic part is played only by the senior component (dominant mode) of the wave spectrum (so-called zero mode). At nighttime, the number of important modes increases. For example, at $f \approx 15$ kc/s, it is necessary to take into account more than 5-7 modes; at a frequency of 30 kc/s, more than 10-12; there are even 3 or 4 modes at $f \approx 5$ kc/s.

The necessity of taking into account the large number of terms of the sum which determines the interference multiplier makes computation of the field for nighttime more difficult. Another difficulty lies in the fact that it is more complicated to compute the true coefficient of reflection from the ionosphere for nighttime. This is conditioned by the fact that in the daytime the coefficient of refraction has a simpler form and depends only on one parameter, N/ν, so that it can be described in the range of low frequencies, with which we have been dealing, with the aid of formula (30.9). The course of N/ν under various conditions is close to the curve shown in Fig. 248. Therefore, as long as the frequency is not excessively small (so that the length of the wave is on the order of, or smaller than, 200-250 km, -- the thickness of the lower half of the ionosphere), and the leakage through the ionosphere of waves at low frequency is not large, it is possible to approximate the ionosphere by a transition layer. This assumes $(N/\nu) \approx$ a constant greater than 89-90 km. This reduces computation considerably.
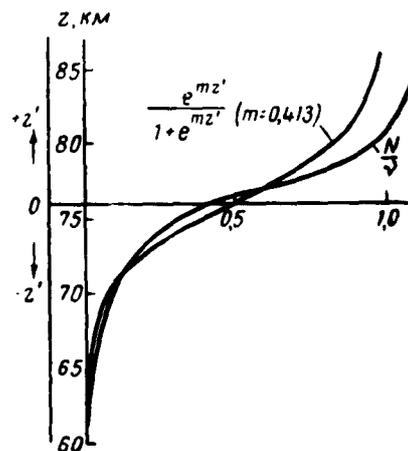
Fig. 248 - Dependence of N/ν on the altitude in the lower part of the ionosphere in daytime. The curve is drawn which closely approximates

$$\frac{N}{\nu}(z)$$

Curves of the field of strength, as a function of frequency for various distances (Figs. 249-252), were computed with the aid of such a model of the ionosphere by the method described above. In computations, it was assumed that the coefficient of reflection from the earth is equal to one, i.e., $\sigma = \infty$. The values of S_n and $P(C_n)$, obtained for the given model of the ionosphere, are given in Table 37, as are also values of $(N/\nu)_{eff}$, which characterize the effective conductivity of the lower region of the ionosphere at various frequencies. In computation, a number of modes were used whose corresponding values, S_n and $P(C_n)$, are given in Table 37.
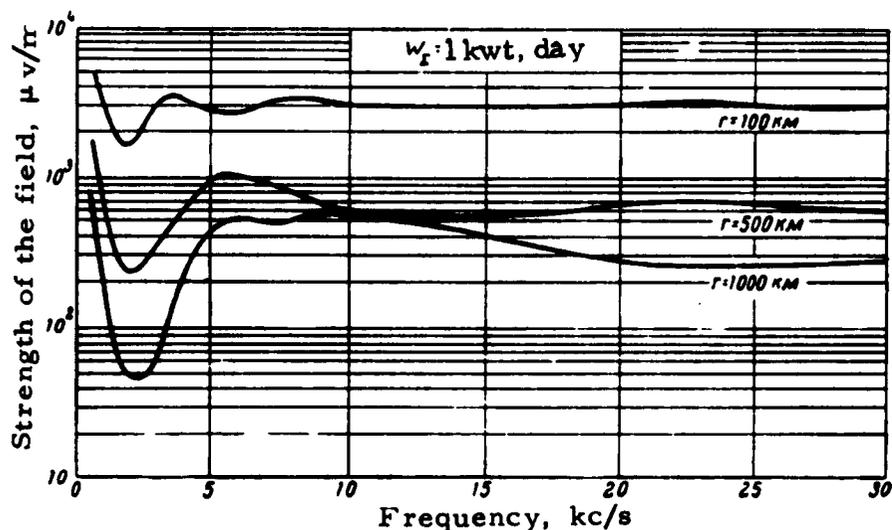

Fig. 249 - Curves of field strength at daytime in the range 500-30,000 kc/s on distances 100, 500 and 1000 km.

Table 37

Values S_n, $P(C_n)$ and $(N/\nu)_{eff}$ of various modes for the daytime model of the ionosphere.

| f, c/s	$(N/\nu)_{eff}$	S_{n1}	$-S_{n2}$	$\|P(C_n)\|$	arg $\{P(C_n)\}$
			$n = 0$		
500	$7 \cdot 10^{-5}$	0,060	0,069	0,488	0°54
1 000	$3 \cdot 10^{-5}$	1,055	0,096	0,451	7 17
3 000	$1 \cdot 10^{-5}$	0,9544	0,0499	0,9764	30 43
5 000	$4 \cdot 10^{-6}$	0,9830	0,0079	0,9275	13 57
7 000	$3 \cdot 10^{-6}$	0,9915	$3,089 \cdot 10^{-3}$	0,8830	9 46
10 000	$2 \cdot 10^{-6}$	0,9959	$1,211 \cdot 10^{-3}$	0,8713	8 49
15 000	$1 \cdot 10^{-6}$	0,9983 / 0,9978	$5,65 \cdot 10^{-4}$ / $8,321 \cdot 10^{-4}$	0,8478 / 0,8256	9 25 / 10 56
20 000	$7 \cdot 10^{-7}$	0,9990 / 0,9985	$3,15 \cdot 10^{-4}$ / $7 \cdot 10^{-4}$	0,8410 / 0,7820	9 32 / 13 00
30 000	$5 \cdot 10^{-7}$	0,9996 / 0,9993	$1,358 \cdot 10^{-4}$ / $3,075 \cdot 10^{-4}$	0,8456 / 0,7940	9 07 / 12 39
			$n = 1$		
5 000	$4 \cdot 10^{-6}$	0,8685	0,1088	0,7175	28 09
7 000	$3 \cdot 10^{-6}$	0,9235	$3,881 \cdot 10^{-2}$	0,8752	15 04
10 000	$2 \cdot 10^{-6}$	0,9621	$1,276 \cdot 10^{-2}$	0,9014	12 05
15 000	$1 \cdot 10^{-6}$	0,9840 / 0,9800	$6,183 \cdot 10^{-3}$ / $9,30 \cdot 10^{-3}$	0,9116 / 0,9000	12 31 / 13 30
20 000	$7 \cdot 10^{-7}$	0,9911 / 0,9800	$3,48 \cdot 10^{-3}$ / $7,8 \cdot 10^{-3}$	0,9195 / 0,9000	12 26 / 13 18
30 000	$5 \cdot 10^{-7}$	0,9960 / 0,9980	$1,47 \cdot 10^{-3}$ / $3,31 \cdot 10^{-3}$	0,9147 / 0,8600	11 52 / 15 00
			$n = 2$		
10 000	$2 \cdot 10^{-6}$	0,8880	$5,194 \cdot 10^{-2}$	0,9233	13 17
15 000	$1 \cdot 10^{-6}$	0,9512	$2,011 \cdot 10^{-2}$	0,9783	10 13
20 000	$7 \cdot 10^{-7}$	0,9730	$1,095 \cdot 10^{-2}$	0,9818	9 17
30 000	$5 \cdot 10^{-7}$	0,9878	$4,583 \cdot 10^{-3}$	0,9880	8 44
			$n = 3$		
20 000	$7 \cdot 10^{-7}$	0,9418	$2,20 \cdot 10^{-2}$	0,9930	7 22
30 000	$5 \cdot 10^{-7}$	0,9742	$8,88 \cdot 10^{-3}$	1,0001	6,23
			$n = 4$		
30 000	$5 \cdot 10^{-7}$	0,9551	$1,412 \cdot 10^{-2}$	1,0002	5 16
			$n = 5$		
30 000	$5 \cdot 10^{-7}$	0,9302	$2,041 \cdot 10^{-2}$.	0,9977	4 46

Fig. 251 - Curves of the field strength in the range 500-30,000 c/s on distance 5000 km.

Fig. 250 - Curves of the field strength in the range 500-30,000 c/s on distances 1000, 2000 and 3000 km.

$w_{\Gamma}=$ 1 kwt, day, r = 10,000 km
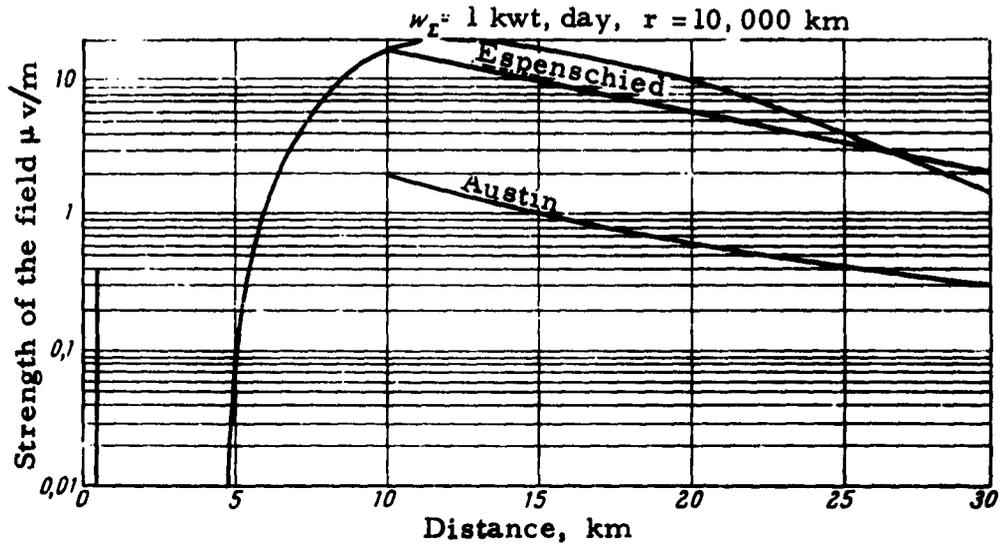
Fig. 252 - Curves of the field strength in the range 50-30,000
c/s on distance of 10,000 km.
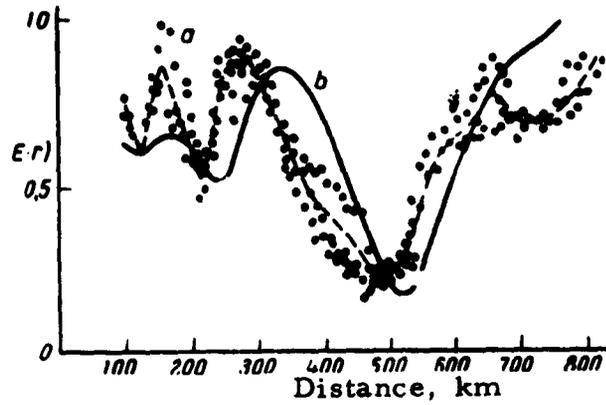
Fig. 253 - Results of measurements (a) field strength on 16
kc/s, measured in an aircraft over England.
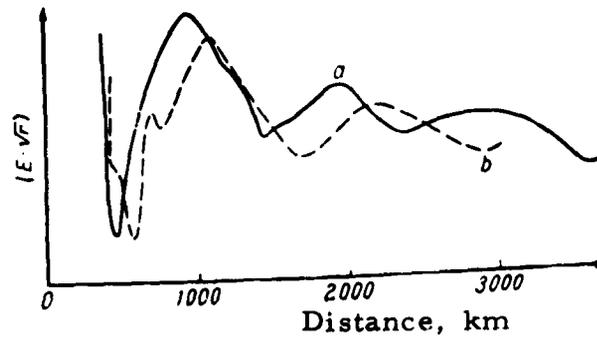
Fig. 254 - Results of measurements (a) and computations (b)
of the field strength on the frequency 16 kc/s,
measured in an aircraft on the route England-Cairo.

Examining Figs. 249-252, which show only average curve of field strength (the separate oscillations are smoothed out), indicates that the curve of field strength, as a function of frequency, is characterized by the following peculiarities: For a constant intensity of the source (as a function of frequency), over the range 7-14 kc/s, the amplitude has a smooth maximum for all distances. After this, it slowly decreases. With increase in distance, the maxima are displaced towards greater frequencies. A sharp minimum of amplitude of the field occurs at frequencies between 2-3 kc/s. Its depth increases strongly with distance, so that

$$\frac{E_{max}}{E_{min}} \approx 10 \text{ at } r \approx 1000 \text{ km, and } \frac{E_{max}}{E_{min}} \approx 2 \cdot 10^3 \text{at } r \approx 3000 \text{ km.}$$

Besides, there is a second maximum amplitude in the ultralong frequency region, as shown in the following paragraph.

Comparing the results of various measurements with computations gives rather good agreement, if the given model of the ionosphere is used, even without detailed precise definition of the value of (N/v) in order to conform to conditions of the experiment. This is evident from Figs. 253 and 254 which show the results of measurements and computations of the field tension at a frequency of 16 kc/s. Naturally, for a closer analysis of the results of measurements, it is necessary to know the dependence of N/v on altitude.

3. Phase and Velocity of Radio Waves

Analysis of the argument of the complex interference multiplier shows that phase also changes in an irregular way with distance near the source, and then, asymptotically, tends to the linear factor $(k_o S_{o1} r)$, when all waves are damped except the zero mode. The corresponding regularities are evident from Fig. 255, which shows the course of the supplemental phase

$$\varphi = \Phi - \frac{\omega r}{c} \tag{30.10}$$

for the frequency, 15 kc/s, and for two values of $(N/v)_{eff}.$[†]

[†]

In Fig. 255, apparently, he has plotted $-\varphi$ (JRW)
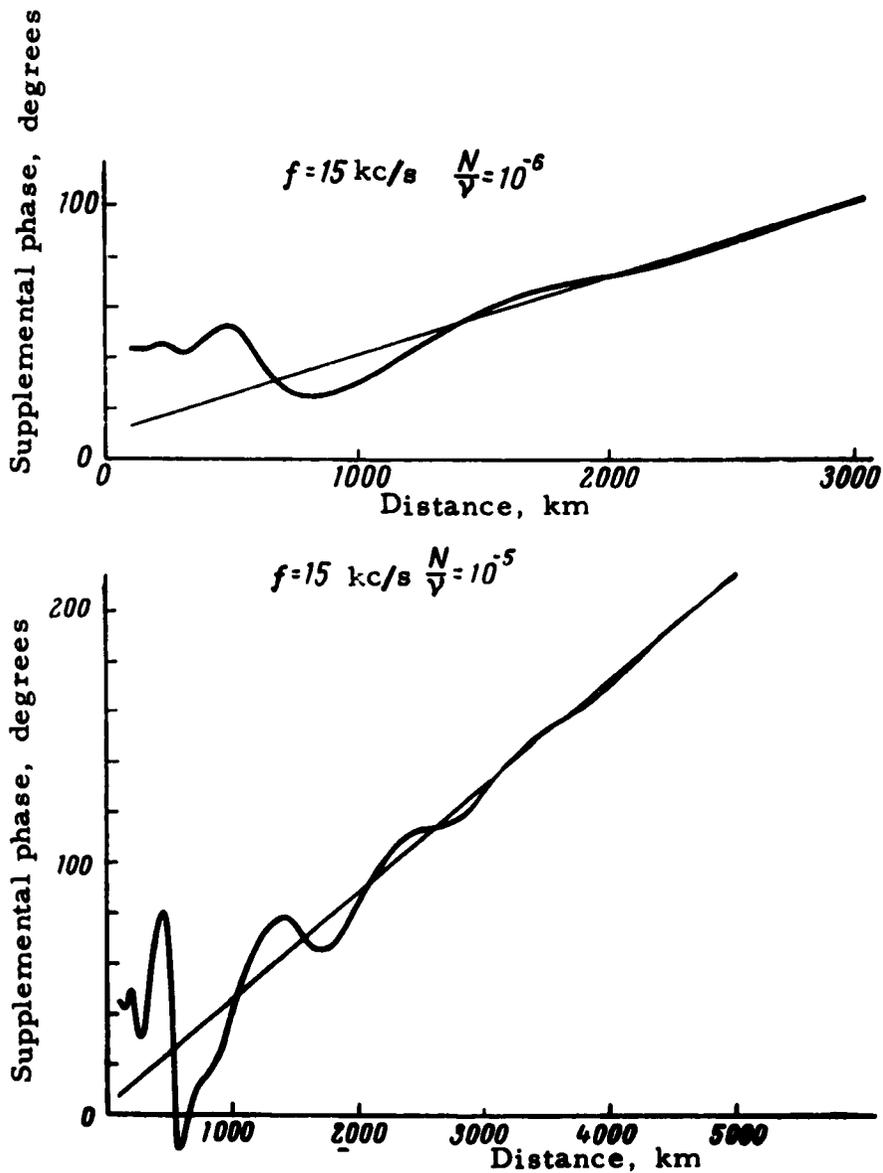
Fig. 255 - Dependence of supplemental phase on distances on frequency 15 kc/s for two different values of N/ν.

Naturally, the differential phase velocity also changes in a similar way

$$v = \frac{c}{1 + \frac{c}{\omega} \frac{\partial \varphi}{\partial r}} = \frac{\omega}{\frac{d\Phi}{dr}} \qquad (30.11)$$

(Fig. 256), and the average phase velocity

$$v = \frac{c}{1 + \frac{c}{\omega} \frac{\Phi}{r}} \qquad (30.12)$$

(Fig. 257). Differential velocity deflects in an irregular way for different distances from the radiator. It may be larger or smaller than c, and it departs from c by 10 to 15 percent and more. On the other hand, the average phase velocity is always greater than c by 3 to 4 percent near the source, and, if far from it, only by a fraction of a percent. In the same manner as the amplitude of the field, the curve of the phase and, consequently, the velocity, changes in detail with small changes of ionospheric parameters or frequency, while preserving its general character (see Figs. 255-257).

Using the same model of the ionosphere, the dependence of average phase velocity of radio waves on frequency for various distances from the radiator is computed. The corresponding dependences are given in Figs. 258-260. Here, considering the small difference between $\bar{v}$ and c, values are derived of $(\bar{v}/c - 1)$. From these figures, it is evident that average phase velocity is everywhere larger than c, and decreases monotonically in the range of frequencies above its maximum, between 2 and 3 kc/s $(\bar{v} - c) > 0$. From the reasoning in the following paragraph, where experimental values (obtained from the analysis of the spectrum of atmospherics) are compared to theoretical values of $\bar{v}/c$, it follows, however, that at $f \lesssim 2$ kc/s, there is a discrepancy. It is supposed [171] that this discrepancy depends on the fact that, at these frequencies, the selected model of the ionospheric transition layer becomes unsuitable since the wavelength approximates the thickness of the whole lower ionosphere

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Al'pert, Ya. L., and S. V. Borodina, An investigation of long and very long radio wave propagation using analysis of atmospherics, Radioengineering and Electronics, 1, 293 (1956).

and more. Also, the influence of leakage of waves, excited in the
earth-ionosphere waveguide, into outer space beyond earth limits,
is felt. Such leakage is already noticeable at frequencies between
7 and 10 kc/s. This is the condition which obtains in the case of the
appearance of the so-called whistling atmospherics, described in
the following paragraph. Up to now, sufficiently extended calculations
have not existed for the propagation of electromagnetic waves at low
frequency which took into account this semi-transparency of the
ionosphere.
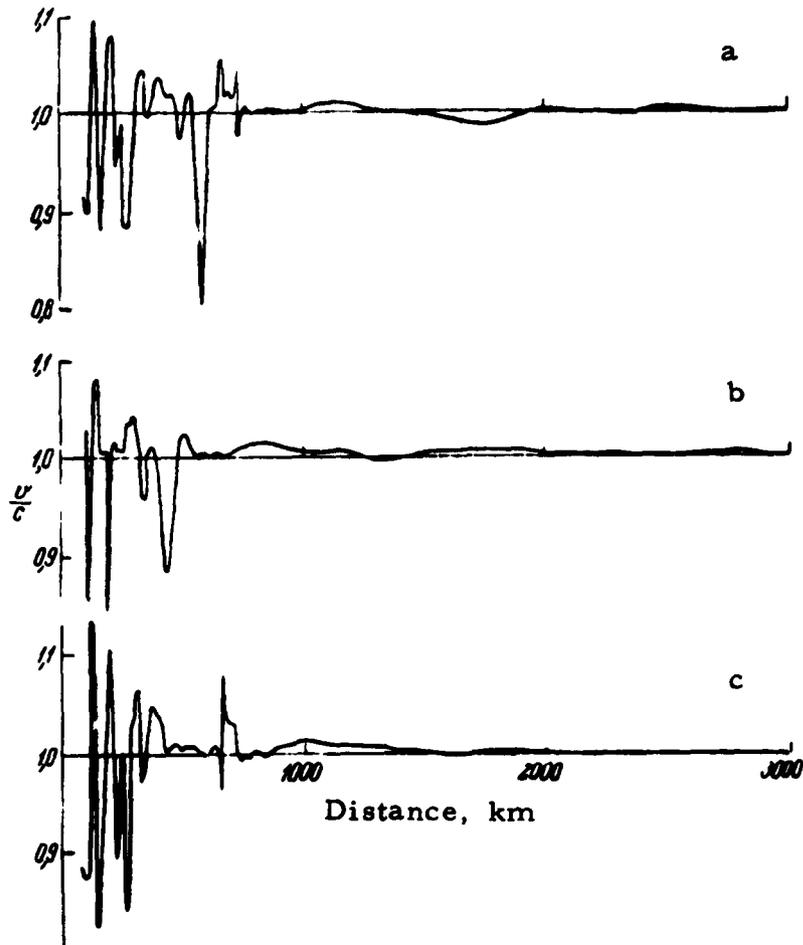
Fig. 256-Dependence of relation of differential
phase velocity v to c on the distance at a fre-
quency of 15 kc/s for three different values of
N/ν.
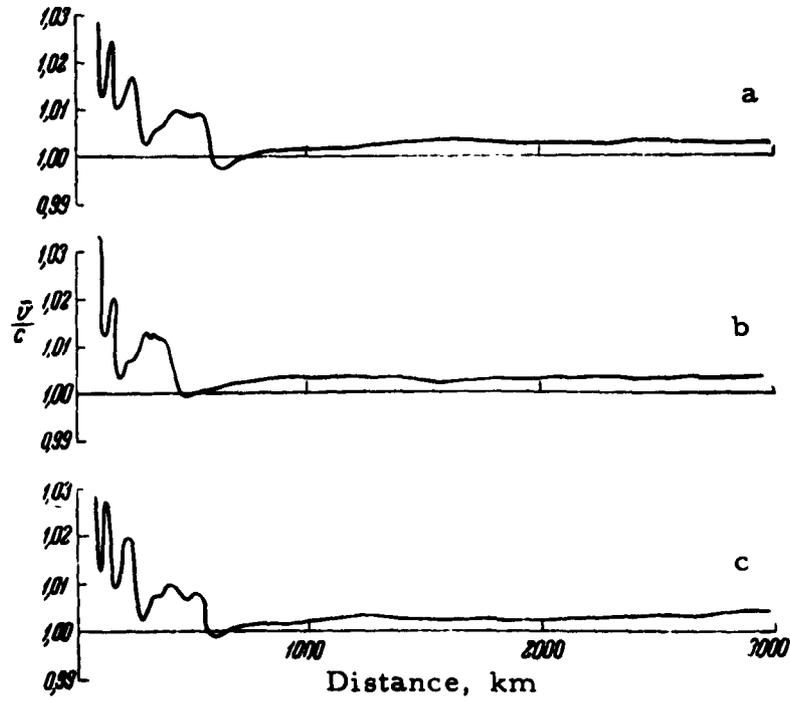
Fig. 257 - Dependence of relation of average phase velocity $\bar{v}$ to c on distance for frequency 15 kc/s for three different values of N/ν.
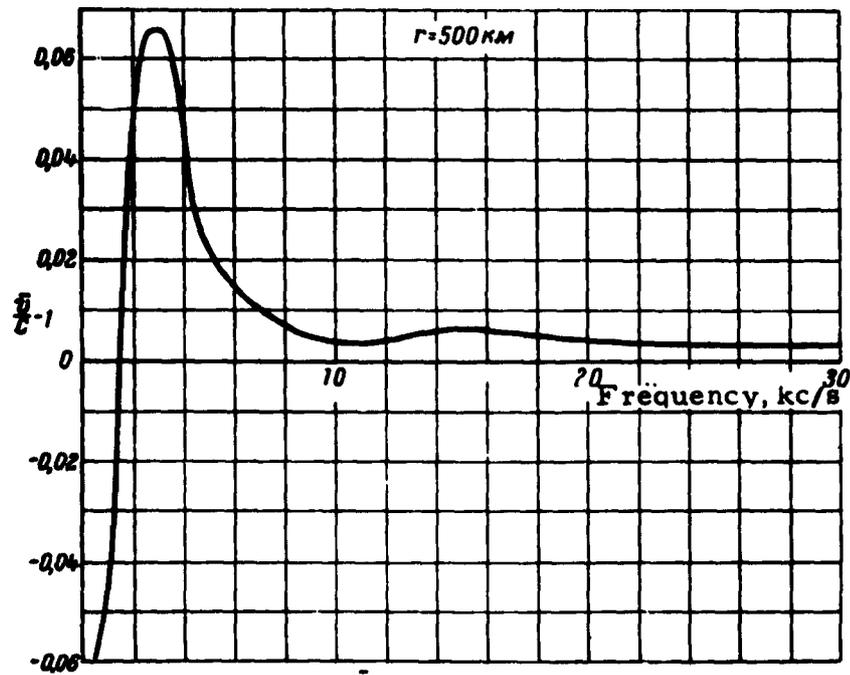
Fig. 258 - Dependence of $(\frac{\bar{v}}{c} - 1)$ with frequency on distance 500 km.
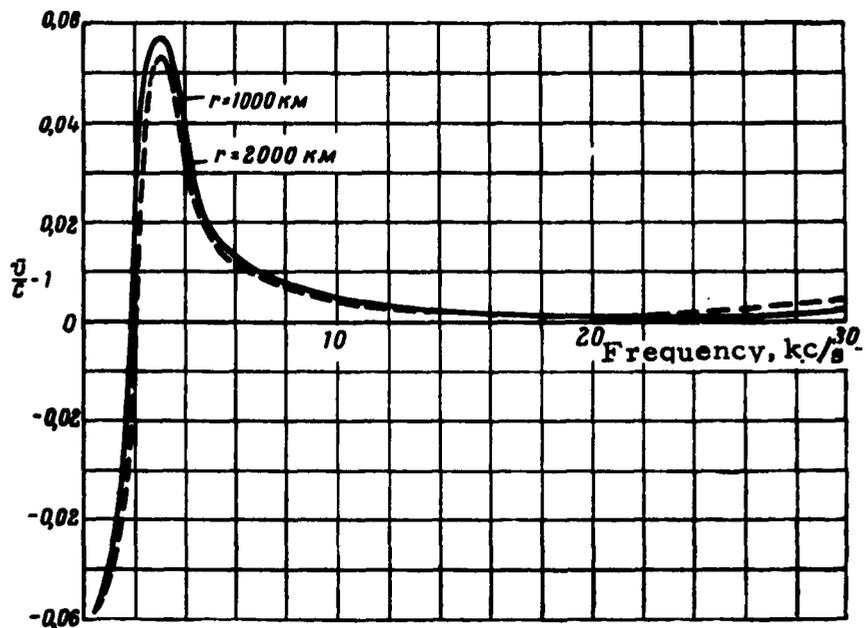
Fig. 259 - Dependence of $(\frac{\bar{v}}{c} - 1)$ with frequency on distance 1000 and 2000 km.
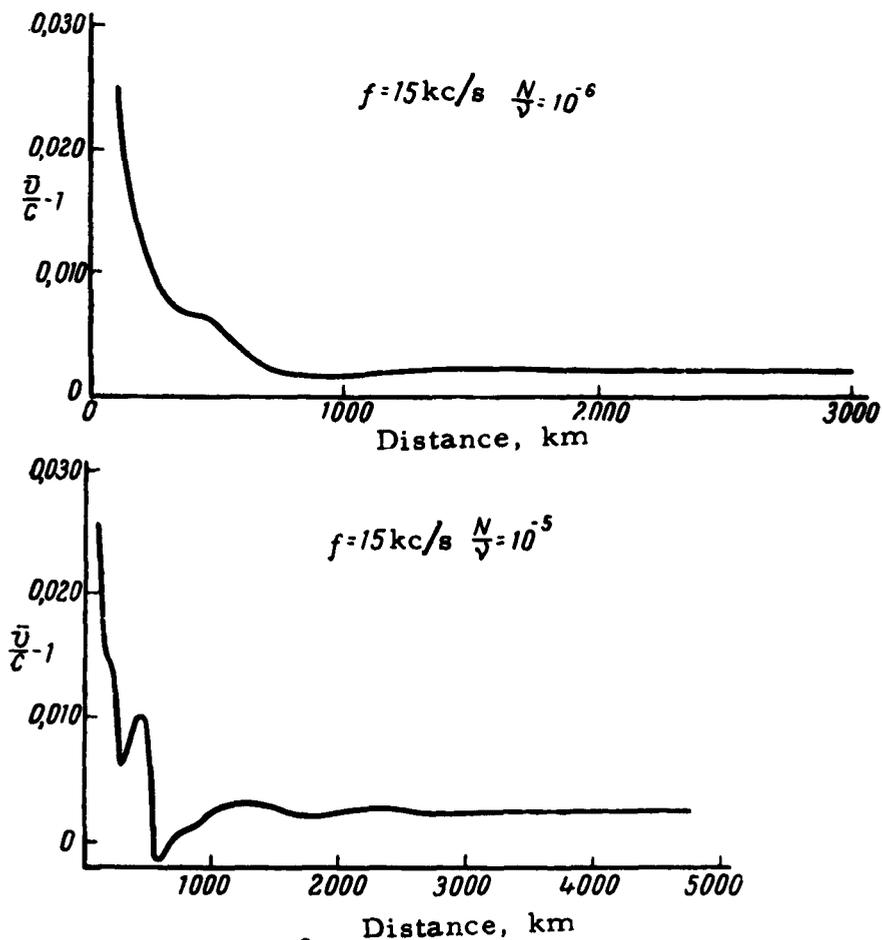
Fig. 260 - Dependence $(\frac{\bar{v}}{c})$ on distance with frequency of 15 kc/s for two values of N/v.

31. Electromagnetic Waves of Ultralow Frequency

This range of electromagnetic waves, created by various radio-arrangements, is limited to long radio waves from 20 to 25 km long (15-12 kc/s). However, a source exists in nature, radiating a continuous, especially intense spectrum of waves, beginning with frequencies from 30 to 50 kc/s, and extending down to several hundred c/s and less. This source is lightning discharges. The waves excited by them are received over long distances. It is possible to record several such signals (called atmospherics) a minute, almost continuously at every point of the globe. The investigation of atmospherics permits us to obtain a series of data on the propagation properties of the waves forming a part of their spectrum. Using this very method, it was possible to determine the average velocity vs. time of electromagnetic waves in the range from 3-20 kc/s [172].

The analysis of atmospherics has also permitted us to show that the minimal losses near the earth are suffered by waves between 25-30 km, and to obtain valuable data on the properties of the lower ionosphere [173] which play an important part in propagation of the whole range of radio waves. Finally, based on theoretical analysis of the form of atmospherics, we can check to what degree methods of computation of long waves (currently in use and described above), are adequate tools for the treatment of the physical substance of the phenomena over a wide range of low frequencies [173]. In line with this, experimental investigations of some varieties of such signals - the so-called whistling atmospherics - taken in conjunction with the theory of their propagation, will permit us to estimate the leakage of long waves through the ionosphere, and to obtain the properties of the outer ionosphere and interstellar gas. It also will allow us to obtain important information about the shortest waves in this range. Thus, the study of propagation of electromagnetic waves, excited by lightning discharges, is closely linked at present with investigations

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    Al'pert, Ya. L., and S. V. Borodina, On the propagation velocity of audio frequency electromagnetic waves, Radio Engineering and Electronics, 4, 195 (1959).
173
    Al'pert, Ya. L., and D. S. Fligel, A synthesis of atmospherics and the effective parameters of the lower part of the ionosphere at low frequencies, Radio Engineering and Electronics, 4, 202 (1959).

of the propagation of radio waves. Below, in brief, are given some properties and results of investigations of atmospherics, which are of interest from exactly this point of view.

1. Atmospherics

An atmospheric is a signal, formed by the spectrum of radiated lightning, at a sufficiently large distance from it. Many near-field observations show that among various forms of discharges of lightnings, the most frequently registered signals are of the type presented in Fig. 261 a, b; while at greater distances, exceeding 200-300 km approximately, in 75 percent of the cases, the received signals are of the type shown in Fig. 261 c, d.

It is established, that for various computations, at frequencies seemingly exceeding 1-2 kc/s, it is possible to select the source of atmospherics of more or less standard form, described by a temporary function

$$E(t, 0) \approx e^{-\cdot \, at} - e^{-\beta t} \tag{31.1}$$

and, consequently, having the spectral density[†]

$$A(\omega, 0) \, e^{i \psi (\omega, 0)} = \frac{\beta - a}{\sqrt{(a^2 + \omega^2)(\beta^2 + \omega^2)}} \, e^{\arctg \{ - \frac{\omega(\beta + a)}{a \beta - \omega^2} \}} \tag{31.2}$$

with the values of $a \approx 4.4 \cdot 10^4$ and $\beta \approx 4.6 \cdot 10^5$ [241]

[†]
  To be consistent, $i \psi (\omega, 0)$ in (31.2) should be prefixed with a negative sign (JRW)

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  Bruce, C. E., and R. H. Golde, The lightning discharge, Journ. IRE, 88, 487 (1941); Lightning Research, 2, 12 (1949).
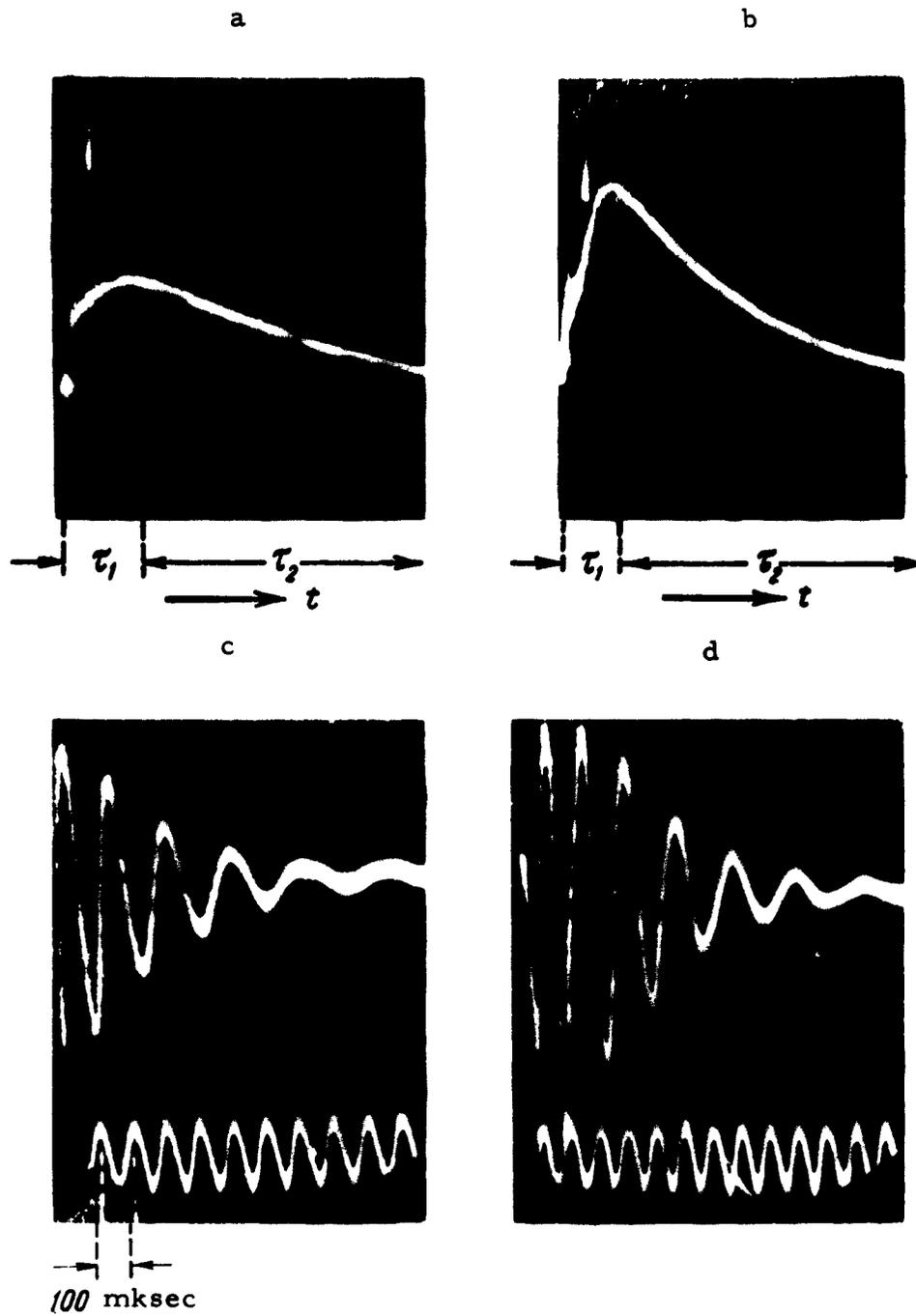
Fig. 261 - Photo-oscillograms of lightning discharges at near
distances (a and b) and signals excited by them (c and d) on
large distances. The sinusoid on the lower part of c and d
is drawn for a time scale. (mksec = micro sec)

Temporary function, $E(t, r)$, describing the form of the atmospheric itself, is the Fourier's integral

$$E(t, r) = \frac{1}{2\pi\sqrt{r}} \int_{-\infty}^{\infty} \{B(\omega, r) A(\omega, 0)\} e^{-i\{\Phi(\omega, r) + \psi(\omega, 0)\}} e^{i\omega t} d\omega ,$$

$$(31.3)$$

where $B(\omega, r) e^{i\Phi(\omega, r)}$ is the interferential multiplier[†] obtained above [see (30.7)], or, alternatively speaking, a function of the passage of an atmospheric through the waveguide.

Naturally, for the study of the observed experimental data, it is expedient to approach signals, $E(t, r)$ from two aspects. First, it is possible to set a general condition of study in summary form over a wide range of frequency for the propagation function, $B(\omega, r) e^{-i\Phi(\omega, r)}$. For this, it is necessary to perform a theoretical synthesis of waves propagating from the lightning discharge by computing the integral (31.3), and checking to what extent the theoretical form of the signals, $E(t, r)$, coincides with the experimental one. Such investigations should show the general deficiencies of the theory.

Secondly, it is also possible to solve the reverse problem on the basis of harmonic analysis of $E(t, r)$; that is, by determining the linked function, (31.3), to find the interference multiplier from the observed signal[††]

$$B(\omega, r) e^{-i\Phi(\omega, r)} = \frac{A(\omega, r)}{A(\omega, 0)} e^{i\{\psi(\omega, r) - \psi(\omega, 0)\}} , \qquad (31.4)$$

where

$$A(\omega, r) e^{i\psi(\omega, r)} = \int_{o}^{\infty} E(t, r) e^{-i\omega t} dt .$$

In this way it is possible to study the properties of interference multiplier by the analysis of atmospherics. And, from the harmonic

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[†] $e^{i\Phi}$ should be $e^{-i\Phi}$ (JRW)

[††] In the right-hand side of (31.4) e^{i} should be e^{-i} and in the following equation $e^{i\psi}$ should be $e^{-i\psi}$ (JRW)

analysis of $E(t, r)$, we are able to determine the phase, $\psi(\omega, r)$, with precision to a linear term; that is, only the so-called supplementary phase is obtained. Thus, the phase characteristics, obtained from such analysis, determine

$$\psi(\omega, r) = \Phi(\omega, r) - \frac{\omega r}{c} = \psi(\omega, r) - \psi(\omega, 0) \tag{31.5}$$

and, consequently, the dependence of average phase velocity of electromagnetic waves at a given frequency over a given distance [172].

Further analysis of the results obtained by the method described above leads to the discovery of ionospheric parameters which characterize the propagation of these waves. For instance, given theoretical dependence, the velocity of radio waves may be obtained from these parameters.

This shows us that the proposed course of investigation has been carried out in full. First, theory is checked by comparison with experiments. Then, physical values are determined from the experiments which characterize the propagation of the waves under consideration. Finally, if the validity of the theory is proven, then the values obtained by experimental means are again compared with the theoretical dependences of those values of the parameters of the medium, and, in this way, the latter determined. In the quoted papers [168, 171-173], a large part of such a program of investigations

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Al'pert, Ya. L. , On the propagation of low frequency electromagnetic waves over the earth's surface.

171

Al'pert, Ya. L. , and S. V. Borodina, An investigation of long and very long radio wave propagation using analysis of atmospherics.

172

Al'pert, Ya. L. , and S. V. Borodina, On the propagation velocity of audio frequency electromagnetic waves.

173

Al'pert, Ya. L. , and D. S. Fligel, A synthesis of atmospherics and the effective parameters of the lower part of the ionosphere at low frequencies.

was completed. The results obtained, which immediately character-
ize propagation of electromagnetic waves of radio range and ultralow
frequency, are briefly given below.

For the synthesis of wave forms of atmospherics in the range
500-30,000 c/s, the values of wave numbers, $k_o S_n$, (shown above in
Table 37), are utilized. At frequencies smaller than 500 c/s and
greater than 30,000 c/s, the approximate solution of poles (30.4) is
useable for determining $k_o S_n$.

If $5 < f < 500$ c/s, then[†]

$$S_{01} = 1 + \frac{1}{2\sqrt{2}\, zk_o\, \sqrt{\beta}} \ , \qquad S_{02} = -\frac{1}{2\sqrt{2}\, zk_o\, \sqrt{\beta}} \ , \qquad C_0^2 = \frac{e^{-i\frac{\pi}{4}}}{zk_o\, \sqrt{\beta}} \ ,$$

$$S_{n1} = \frac{1}{zk_o\, \sqrt{\beta}\, \sqrt{2(a^2-1)}} \ , \qquad S_{n2} = -\sqrt{a^2-1}\left(1 - \frac{1}{\sqrt{2}\,(a^2-1)\, zk_o\, \sqrt{\beta}}\right) \ ,$$

$$C_n^2 = a^2 + 2C_0^2 \ , \qquad\qquad (31.6)$$

where

$$n = 1, 2, \ldots, \qquad a = \frac{n\pi}{zk_o} \ , \qquad \beta = \frac{4\pi\sigma}{\omega} = \frac{4\pi e^2}{m\,\omega^2}\,\frac{N}{\nu} \ .$$

[†]

Eqns. (31.6) for the S_n's are only valid if $z k_o \sqrt{\beta} \gg 1$, see
reference [242], (JRW).

If $f \lesssim 5$ c/s, then[†]

$$S_{01} = \sqrt{\frac{1 + \sqrt{2}}{2 \sqrt{2} \, zk_o \, \sqrt{\beta}}} \quad , \quad S_{02} = -\sqrt{\frac{\sqrt{2} - 1}{2 \sqrt{2} \, zk_o \, \sqrt{\beta}}} \quad . \tag{31.7}$$

For frequencies where $f > 30,000$ c/s, the coefficient of reflection from the homogeneous ionosphere is well approximated by the formula

$$\rho(C) = e^{-(B_1 + i \, B_2) \, C}. \tag{31.8}$$

Therefore, if B_1 and B_2 are computed by comparing $\rho(C)$ with the true coefficient of reflection of the ionosphere, it is possible to solve the equation of poles simply.

With the help of these formulae, the functions $\{[B(\omega, r) \, A(\omega, r)]\}$, and, $\{[\psi(\omega, 0) + \psi(\omega, r)]\}$, (Figs. 262 and 263), were obtained, and then, by numerical integration, (31.3), signals, $E(t, r)$, computed. It should be kept in mind when evaluating the curves in Figs. 262 and 263, that at frequencies smaller than 1-2 kc/s, these are barely precise, since both the leakage of these waves through the ionosphere, and the spectrum of the source itself at $f \lesssim 1$-2 kc/s, should be taken into account. (See Supplement 7.)

Figure 264 gives the signals, $E(t, r)$, computed in this manner. Their upper part presents photo-oscillograms of atmospherics which were most frequently received at these distances. An exceptionally good correlation between the oscillograms and the results of calculation is plain, tending to establish the fact that the theory apparently takes into account the basic peculiarities of propagation of the spectrum of the waves which form atmospherics.

[†]

Eqn. (31.7) is only valid if $z \, k_o \, \sqrt{\beta} \ll 1$, (JRW).
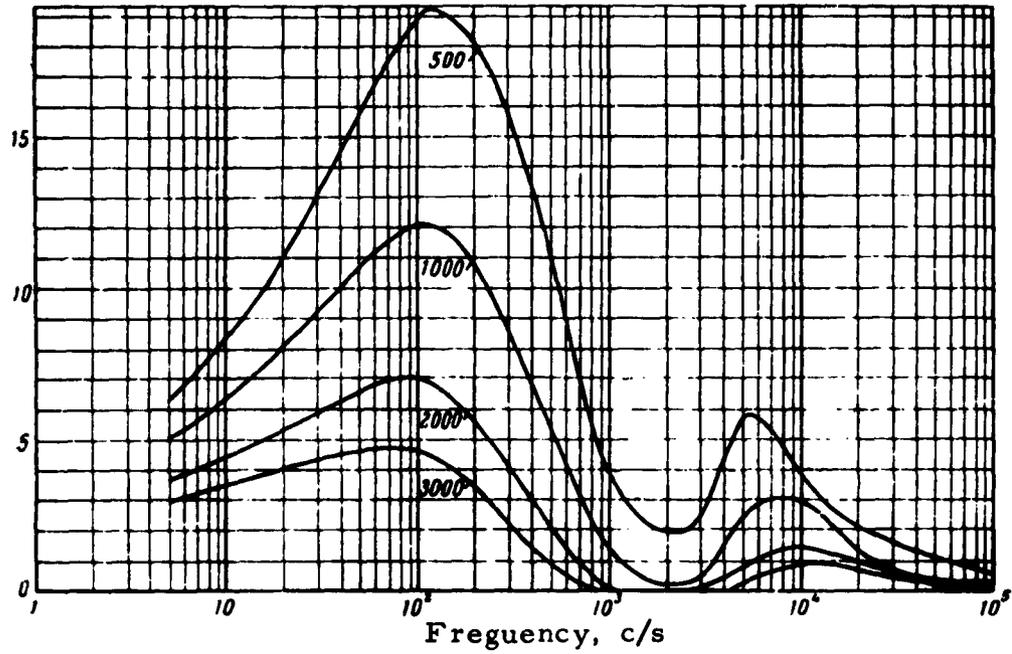
Fig. 262 - Product of moduli of the interference multiplier and the spectral density of the atmospheric at various distances (in km).
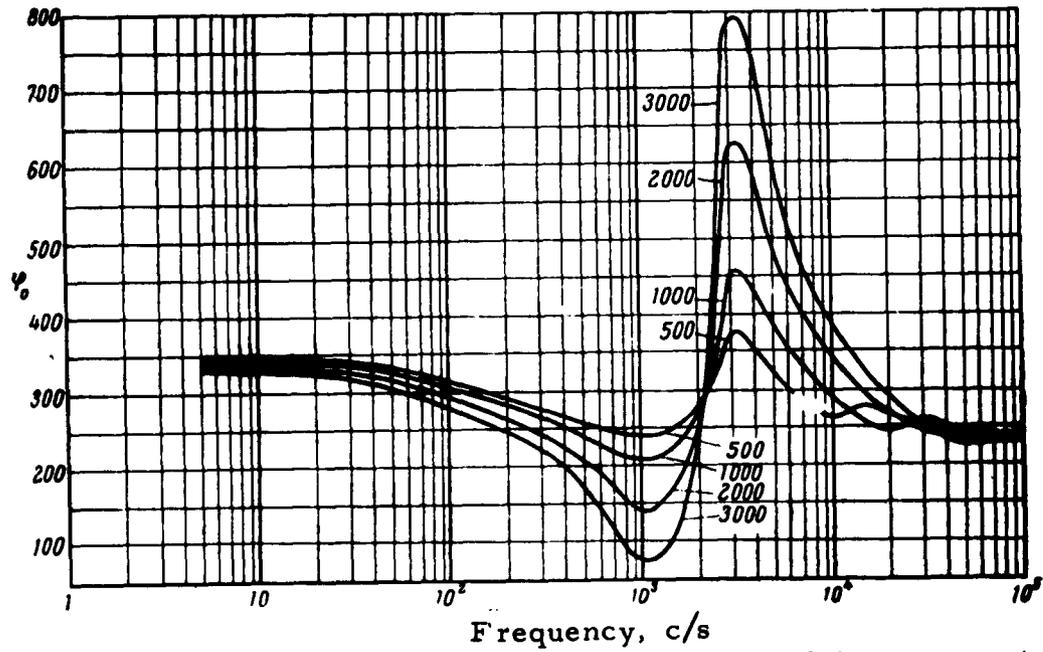
Fig. 263 - The sum of supplemental phase and of the argument of the spectral density of the atmospheric at various distances (in km).
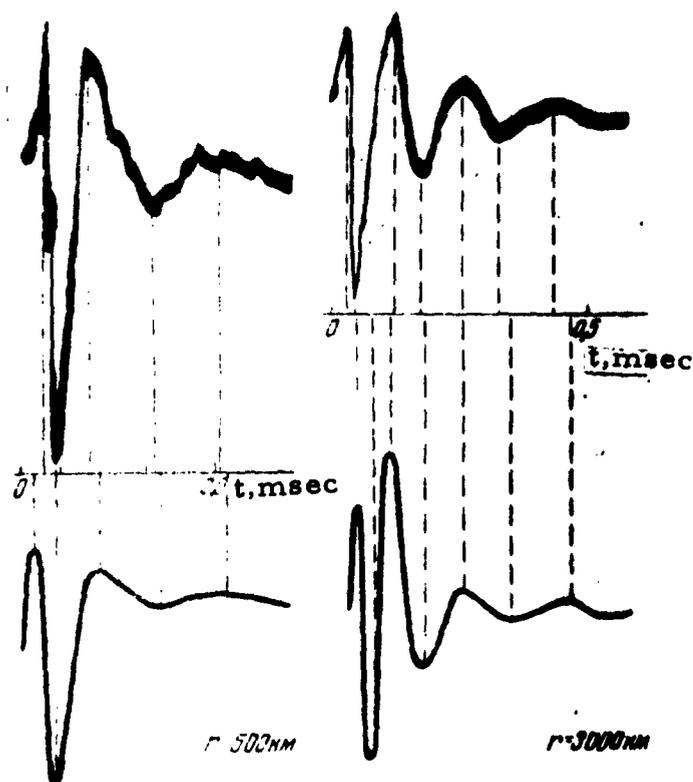
Fig. 264 · Comparison of observed atmospherics with the computed theoretical signals.

Experimental comparison of the modulus of the spectral density of atmospherics with theoretical computations also gives a good correlation (Figs. 265 and 266). However, the most important result of harmonic analysis of atmospherics is the determination of average phase velocity of the electromagnetic waves over the range of low frequencies (Fig. 267). The most frequently encountered experimental values of $\bar{v}/c$, given in Fig. 267a, coincide well with the theoretical values $\bar{v}/c$ up to where frequencies are between 2 and 3 kc/s. At lower frequencies, the theoretical value of velocity gradually decreases and becomes smaller than c, but experiments show a growth in velocity. As mentioned above, this is apparently explained by the fact that leakage through the ionosphere becomes important at very low frequencies. In theory, this is not taken into account.
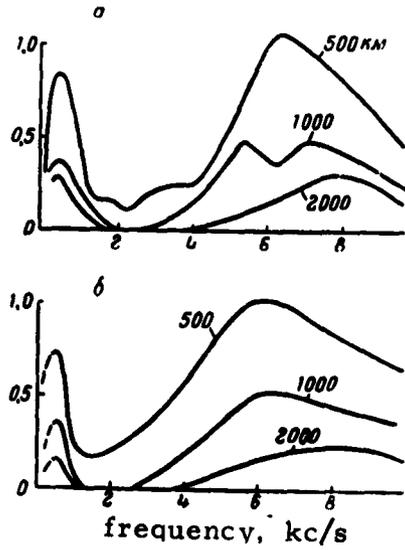
Fig. 265-Experimental (a) and theoretical (b) dependence of relative value of modulus of spectral density of atmospheric on various distances (km) in England.

frequency, kc/s
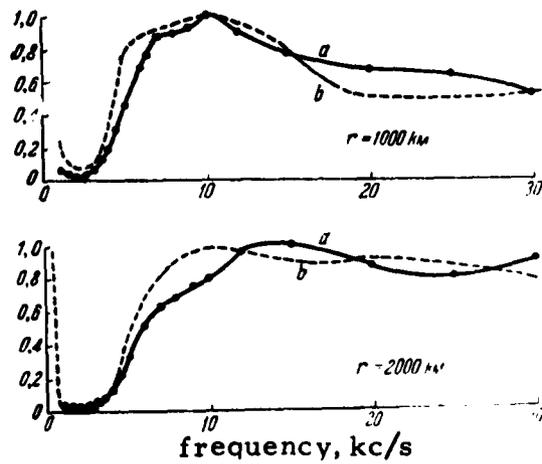
Fig. 266-Experimental (a) and theoretical (b) dependence of relative value of modulus of spectral density of atmospheric on various distances (km). Measurement was conducted in Moscow.

frequency, kc/s

On level
0. 8

0. 5

Most probable values
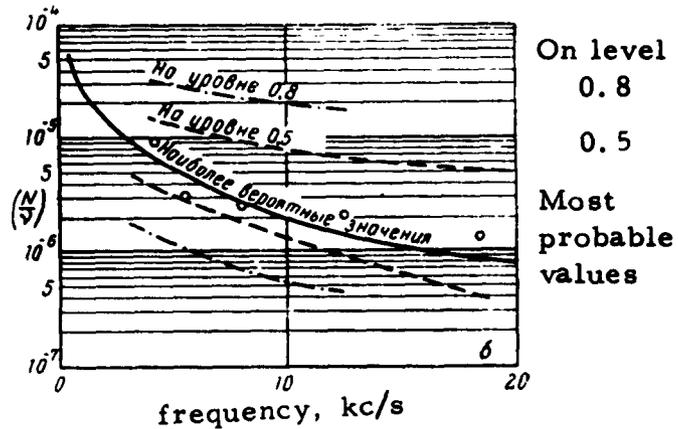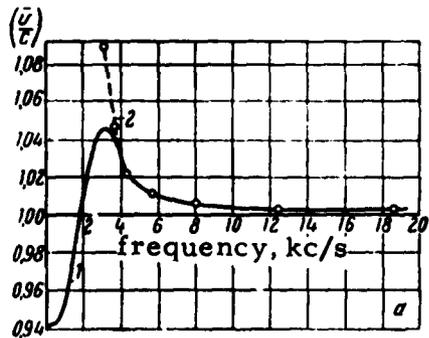
frequency, kc/s

Fig. 267-Theoretical (1) and experimental (2) dependence (v̄/c) and (N/v) on frequency.

Using experimental values of v/c permits us to determine, with the aid of the theoretical dependence v/c on N/v, the values of $(N/v)_{eff}$ observed under real conditions. In the given experiments, the dependence of N/v on frequency, presented in Fig. 267b, was obtained. Naturally, the most probable values of N/v coincided with those given in computations since there is an accord between corresponding values of $\bar{v}/c$.

It should be noted that in the more thorough analysis of signals, $E(t, r)$, which were computed theoretically (see Fig. 264), it appears that for distances of 2000-3000 km and greater, the beginning of the signal is somewhat different than in the observed atmospherics. It is possible that this is linked to the fact that, in the computation of $E(t, r)$, the finite conductivity of the earth and its sphericity were not taken into account. For the spectrum of waves under consideration, this would be important at greater distances and higher frequencies. The higher frequencies are responsible for the beginning part of the atmospheric. In conclusion, it should be noted that the atmospherics investigated here form only one part of these signals, which may be termed the high frequency part, and which have a duration of the order of 0.5 - 1 msec. However, behind this part of the signal there frequently follows a "tail" where duration reaches 10-15 msec. and more (Fig. 268). This part of the signal is formed mainly by waves whose frequencies are less than 500-1000 c/s. At present, theoretical computations of the form of signal in this range have not been carried out to an adequate extent. Also, the "tails" of atmospherics have not been the subject of much experimental study.
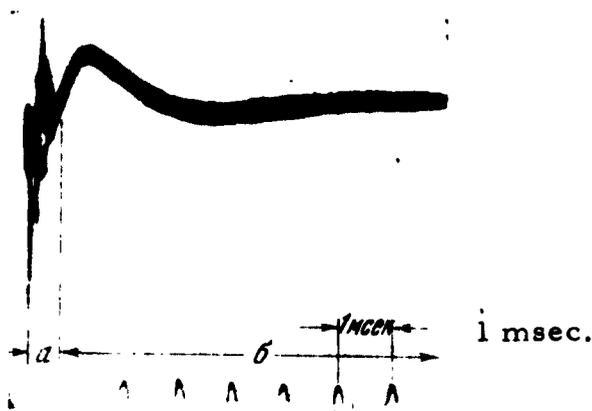

Fig. 268 - Oscillogram of an atmospheric
with a low frequency tail.

2. Whistling Atmospherics

The whistling atmospheric is received by ear as a signal whose frequency changes with time. At first, a low tone is heard; then a high tone. † Most frequently, signals of such type run from 400-8000 c/s [174, 175]. Apparently the greatest intensity of the whistling atmospherics is in the frequency range 3000-4000 c/s.

Records of a single whistling atmospheric and a group of such signals are presented in Figs. 269 and 270. The analysis of such signals has shown that they are associated with lightning discharges originating at points of earth which are connected with the point of observation by a (terrestrial) magnetic line of force (Fig. 271). This type of whistling atmospheric is called a short one. It is late in relation to the lightning discharge with respect to its time of propagation along the trajectory. However, atmospherics are also received which are associated with local thunderstorms. These propagate along a double path; that is, they come back to the point of observation after being reflected from the conjugate point of the earth (Fig. 271). In this case, the signal is called a long one, since the transition over the whole range of frequencies (that is, the whistle), is accomplished within a double time. The time of retardation of the beginning of whistling, in relation to the lightning discharge, is directly determined when the long signals are received. By this method, the time of group retardation of these signals was established as of the order of 2-3 sec. Furthermore, it follows that their trajectory reaches lengths of many thousands of kilometers (see below).

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Usually the high tone precedes the lower tone in conventional whistlers (JRW).

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Storey, L. R., An investigation of whistling atmospherics, Phil. Trans. Roy. Soc., 246, 113 (1953); Can J. Phys., 34, 153 (1956); 35, 1107 (1957).

175

Iwai, A., Outsn, J., On an investigation of whistling atmospherics in Japan, Proc. Res. Inst. Atm., 4, 29 (1956).
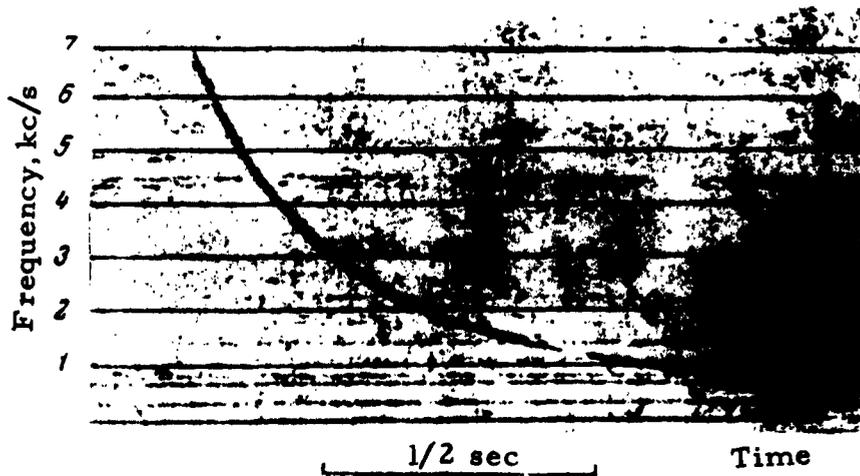
Fig. 269 - Record of whistling atmospheric.
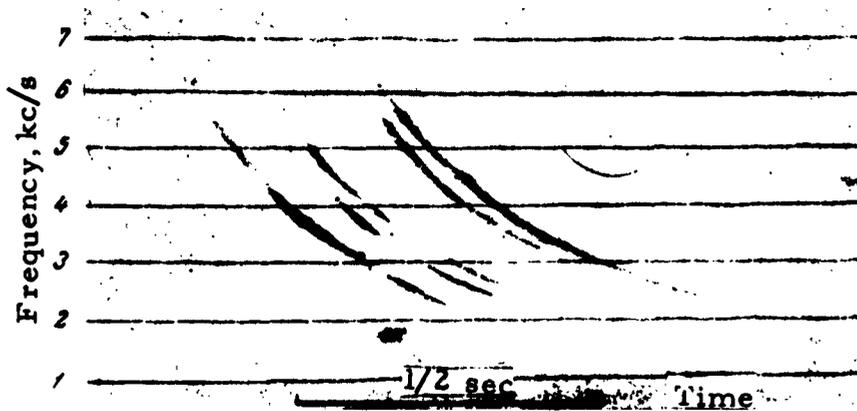
Fig. 270 - Record of a group of whistling atmospherics.
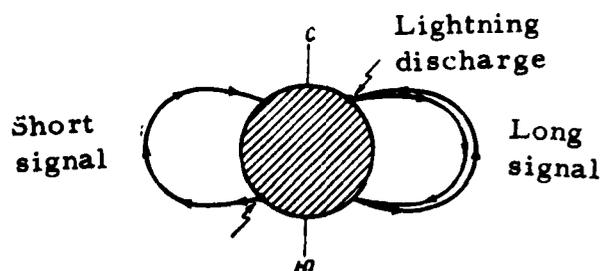
Fig. 271- Schematic presentation of trajectory of the short and long
(whistling) atmospheric.

Experimental data have made it possible to establish that the
change of tone (frequency) of the signal, with time, is well described
by the formula

$$\sqrt{f} = \frac{D}{t} .$$ (31.9)

The quantity, D, is named the dispersion of the whistling atmospheric.
In various cases, for short atmospherics, it changes from approxi-
mately 10 sec to 100 sec and more. Analysis of the sum of the experi-
mental data has shown that the dispersion of long signals is approxi-
mately twice as large as the dispersion of short signals, and that
there is some correlation between probability of appearance of
whistling atmospherics, the state of the atmosphere, and magnetic
activity. This lets us suggest that this observed phenomenon can be
explained in the following way [174]:

As already pointed out in the preceding paragraph, the waves
radiated by a lightning discharge partially leak through the ionosphere,
especially at frequencies between 10^6 - 10^5 c/s. From an analysis of
the coefficient of refraction of the ionosphere, taking into account the
magnetic field of the earth, it follows that this occurs for the quasi-
longitudinal propagation of these waves in the ionosphere. At these
frequencies, when angles as small as 5-10° obtain between the direction
of propagation of the wave with a vector of the magnetic field, H_o,
the coefficient of refraction becomes imaginary. In this way, the
waves passing through the ionosphere are canalized (ducted) along the
magnetic field of the earth, and, with the exclusion of the case where
propagation is precisely longitudinal, are regular waves. (This
special case requires a detailed supplemental analysis for waves of
low frequency.) Given the condition that everywhere along the tra-
jectory

$$\frac{4\pi N_e^2}{m \omega^2} >> 1$$ (31.10)

(neglecting the influence of number of collisions), the coefficient of
refraction of these waves is given by (see 1. 8)):

$$n^2 \approx 1 + \frac{4\pi N_e^2}{m \omega^2 \left(\frac{e H \cos \theta}{m c \omega} - 1\right)} ,$$ (31.11)

174

Storey, L. R. , An investigation of whistling atmospherics.

where θ is the angle between the normal to the wave front and the magnetic field vector H. Computing the time of group retardation of signals

$$t' = \int \frac{c \cos a}{u} \, dS \quad , \tag{31.12}$$

where S is the trajectory of the wave, a is an angle between the group velocity vector and tangent to the trajectory; then

$$u = \frac{c}{n + \omega \dfrac{dn}{d\omega}} \tag{31.13}$$

[see (1.28)], determines group velocity.

Utilizing the group velocity, it is possible to obtain for the case when the longitudinal component of gyroscopic frequency equals

$$\frac{eH}{mc} \cos \theta \gg \omega \quad , \tag{31.14}$$

$$\sqrt{f} = \frac{1}{t} \left\{ \sqrt{\frac{e}{4\pi c}} \int_{(s)} \sqrt{\frac{N}{H \cos \theta}} \cos a \, dS \right\} \tag{31.15}$$

Comparing (31.15) with (31.9), we see that the expression in the braces is the quantity of dispersion, D. If the condition (31.14) is not fulfilled everywhere along the trajectory of the wave packet, then,

$$\sqrt{f} = \frac{1}{t} \left\{ \sqrt{\frac{e}{4\pi c}} \int_{(s)} \sqrt{\frac{N}{H \cos \theta}} \left[\frac{1}{1 - \dfrac{2\pi f m c}{eH}} \right] \cos a \, dS \right\} \tag{31.16}$$

and it is possible to show, in distinction to the first case, when it is possible to utilize formula (31.15), that the time of retardation of the signal does not increase constantly with decrease of frequency. There is an extreme; at some value of frequency, the time of retardation reaches a minimum. On both sides of this value, f, time, t, increases for waves of greater frequency, and also for waves of lesser frequency. It is difficult to detect this effect experimentally, because under usual conditions, the border value of frequency lies in the region of waves which seep through the ionosphere badly. However, here are some experimental proofs of this effect; the extreme may be observed in some series of experiments.

The theoretical explanation of whistling atmospherics given above cannot pretend to be a full description of this phenomenon. To date, there is no sufficiently accurate method of taking into account the influence of ions. This effect may be substantial at lower frequencies in this range. Also, the computations for the trajectory were not made with necessary completeness and precision, nor account taken of anisotropy of the ionosphere, and so on.... Therefore, many important properties of this phenomenon are not made clear. One paper (Fig. 272) [176], indicates that the trajectory of whistling atmospherics should not coincide with the lines of the (terrestrial) magnetic field. Therefore, it follows that the point linked to the source does not lie at the end of the corresponding field line. However, in spite of the absence of theoretical explanation of this phenomenon, it is still possible to obtain, with the aid of the given approximate formulae, some information on density of the medium, path along the magnetic field of the earth, and other quantities along the trajectory of whistling atmospherics. These regions stretch to distances as remote as 10,000-15,000 km from the earth and more (see Fig. 272).

In conclusion, it is expedient to note the following: An attempt was made to ascertain whether effects similar to the whistling atmospherics can be observed through radiation of radio waves. Recently, the results of observations of similar echo-signals at a frequency of 15.5 kc/s [177] were described for the first time. These signals were retarded for approximately 1 sec. relative to the moment of their radiation. From analysis of experimental data, the authors of the quoted paper came to the conclusion that they were watching signals propagating along the line of force of the magnetic field of the earth. Direct proof of the effect of whistling atmospherics with the aid of radio waves was thus obtained for the first time. It is possible to suppose that this phenomenon played a part in various earlier radio

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Maeda, K., and I.A. Kimura, A theoretical investigation on the propagation path of the whistling atmospherics, Rep. Ion. Res. Jap., 10, No. 3, 105 (1956).

177

Helliwell, R.A., and E. Gehrels, Observations of magneto-ionic duct propagation using man-made signals of very low frequency, Proc. IRE, 46, No. 4, 785 (1958).

observations; however, proper attention was not given it.
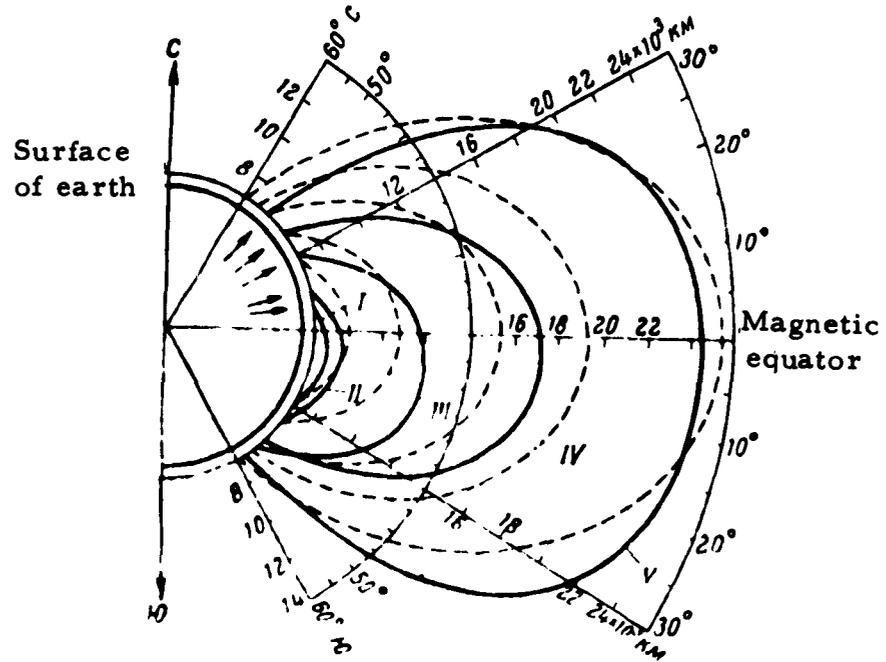
Fig. 272- Trajectories of propagation of whistling
atmospherics (solid lines); magnetic force lines
are presented by dots.

Appendix 6

The Influence of Sphericity of Earth on the Problem of Wave-Directed Propagation of Electromagnetic Waves of Low Frequency

1. In paper [169], the following approximate equation[†] of poles was obtained for the homogeneous spheric earth, characterized by complex coefficient of refraction $\epsilon^*(k_1 = k_o \sqrt{\epsilon^*})$, and homogeneous concentric ionosphere, characterized by the coefficient of refraction, $\rho_2(C)$:

$$i \frac{k_o^2}{k_1^2} \sqrt{k_1^2 - k_o^2} = \left[i k_o C_n + (\frac{S_n}{C_n})^2 (2 R_o)^{-1}\right]$$

$$\times \frac{[1 - \rho_1(C_n) \exp(i 2k_o z C_n)][1 + i(S_n/C_n)^2(2 R_o k_o C_n)^{-1}]}{[1 + \rho_2(C_n) \exp(i 2k_o z C_n)][1 - i(S_n/C_n)^2(2 R_o k_o C_n)^{-1}]} . \quad (D.28)$$

Here, R_o is the radius of earth; z is altitude of the ionosphere; C_n are poles; $S_n = \sqrt{1 - C_n^2}$ and $k_o = \omega/c$. This equation is presented for approximate boundary conditions

$$\frac{d}{dR} [R f_n(R)] = -i \frac{k_o^2}{k_1^2} \sqrt{k_1^2 - k_o^2} [R f_n(R)] \qquad (D.29)$$

when $R = R_o$, and f_n are radial functions, satisfying the equations

$$\frac{d^2}{dr^2} (R f_n) + \left[k^2(R) - \frac{n(n+1)}{R^2}\right] R f_n(R) = 0 .$$

It is easy to be convinced that at $R \to \infty$ (flat earth) the equation (D.28) passes into the equation of poles for the flat earth (30.4).

169

Bremmer, H. National Bureau of Standards Rep. No. 5518 (1957).

†

It should be noted that Al'pert here uses an exp $(-i \omega t)$ whereas, in (D.31) and elsewhere in the book, exp $(i \omega t)$ is employed (JRW).

The equation (D. 28) is rather complicated by itself and is not solved in [169]. However, with its help, S_n and C_n can be expanded into Taylor's series by powers of $1/R$ in the vicinity of $1/R = 0$, to determine the influence of the sphericity of the earth. Then we adopt supplemental components, ΔS_n, ΔC_n, to the values of S_n and C_n for the flat earth, by computation of the corresponding derivatives

$$\frac{\partial C_n}{\partial (\frac{1}{R})} \quad , \quad \frac{\partial^2 C_n}{\partial (\frac{1}{R^2})}$$

and so on, of the right part (D. 28). As a result, one obtains a rapidly convergent series in which it is sufficient to take into account only the first term of the series. Similar computations, given in [243], have shown that in many cases the values, ΔS_n and ΔC_n, are quite large, commensurate with and even larger than, S_n and C_n. Apparently, it is possible, using this method, to compute sphericity of the earth, accurately based in each separate case on those conditions suitable for the formula (D. 28), and using the series for S_n and C_n.

2. In a recently published article [242], a substantial account is given of the present state of the theory of propagation of electromagnetic waves of low frequency. This work also gives the results of a corresponding analysis of the spherical waveguiding problem. When the condition[†]

$$\left(\frac{k_0 \ |\varepsilon_1^*| \ R_0}{2}\right)^{1/3} |C| \ll 1 \ , \tag{D. 30}$$

169
Bremmer, H., National Bureau of Standards Rep. No. 5518 (1957).

243
Borodina, S. V., Yu. K. Kalinin, G. A. Mikhailova and D. S. Fligel, A survey of the present state of investigations of the propagation of very long electromagnetic waves, Radiophysics (Bul. Higher Educational Institutions) 3, 6 (1960).

242
Wait, J. R., Terrestrial propagation of very low-frequency radio waves, a theoretical investigation, J. Research NBS 64D (Radio Prop.), No. 2, 153-204 (Mar. -Apr. 1960).

† In eqn. (D. 30) $<<$ should be $>>$. This was a misprint in reference [242]. For frequencies in the range 8-30 kc/s this condition is usually violated for the dominant modes (JRW).

is fulfilled, the electromagnetic field can be computed with the aid
of formula (30.1), or (30.6), with the multiplier, $\theta/\sin\theta$, of geo-
metric character, which takes into account the correction for
sphericity (see above). Here, however, the poles, C_n, and corres-
pondingly, wave numbers, $k_o S_n$, are determined from an equation
of poles of the form

$$\rho_1(C_n)\,\rho_2(C_n')\,\exp\left\{-i\,2k_o\int_0^z\left[C_n^2+\frac{2x}{R_o}S_n^2\right]^{\frac{1}{2}}dx\right\}=e^{-2\pi i n}\quad,\quad(D.31)$$

where, supplementing designations used in par. 30

$$C_n'=\sqrt{1-S'^2}\quad,\quad a\ S'=\frac{R_o S}{(R_o+z)}\quad.$$

The coefficients of reflection from the earth and the ionosphere
are described as before with the aid of Fresnel's formulae. When
computing the general solution of the spherical problem, with a high-
er degree of precision, when limitation (D.30) is not fulfilled, the
equation of poles has a considerably more complicated form. This
is not treated here; refer to paper [242]. Also, there is no extended
comment on the results of analysis of the corresponding solution of
the waveguiding problem for the region close to the antipode [9, 163,
242]; nor is there an examination of the effect of the magnetic field
of the earth [167, 242]; and many other questions.

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 Wait, J.R., Terrestrial propagation of very low-frequency radio
 waves, a theoretical investigation.
  9
 Al'pert, Ya. L., On the propagation of low frequency electromagnetic
 waves over the earth's surface, Acad. Science, USSR (1954); Progress
 in Physics, 60, No. 3, 360 (1956)
163
 Schumann, W., Uber die Oberfelder bei der Ausbreitung langer
 elektrische Wellen in System Erde-Luft-Ionosphare.
167
 Budden, K.G., The propagation of a radio atmospheric.

Appendix 7

Passage of Electromagnetic Waves of Low Frequency
Through the Ionosphere

For the purpose of computing the influence of leakage of electro-
magnetic waves through the ionosphere (upper wall of the waveguide),
it is necessary to compute more accurately the coefficient of re-
flection of electromagnetic waves of low frequency from the iono-
sphere, $\rho_2(C)$, taking into account the influence of the magnetic field
of earth. This is very important for the lower frequencies under
consideration, and is necessary to perform for both the spherical
and the plane problem. Such computation, which is somewhat com-
plicated, is not realized practically in any of the quoted works. How-
ever, it is expedient to point out here, there is a possibility of making
a more simple computation of poles, based on the physical substance
of the corresponding solution of this problem, presented in Sec. 30.

For this purpose, it is necessary to solve the equation of poles,
introducing instead of the coefficient of reflection, $\rho_2(C)$, the
quantity

$$\rho_2(C) \left[1 - \exp i\, k_o \int_o^\infty (n - i\varkappa)\, dz \right] \ , \qquad (D.32)$$

where

$$\exp i\, k \int_o^\infty (n - i\varkappa)\, dz$$

is a function characterizing the transmission into the ionosphere of
waves of the corrresponding frequency.

Since only the extra-ordinary wave in quasilongitudinal propaga-
tion passes through the ionosphere, it is possible to use the co-
efficients of refraction and attenuation, n, and, $\varkappa$, given above in
Supplement 1 [see (D.6)], for compu·ation of (D.32). The results of
computations of the transmission function of the ionosphere for a real
model of the upper atmosphere (s⸱e Sec. 6) are given in Table D;
separately for day and night. Corresponding numerical integration is
conducted to the point where z = 3000 km (upper limit of integration)
which is sufficient for these evaluations. In Table 4 values are given

$$\exp - \frac{\omega}{c} \int_o^{3000} \varkappa\, dz$$

and the relation v/c, of average phase velocity of wave v to c, characterizing modulus and argument of function for passage through the ionosphere by ordinary (index +) and extra-ordinary (index -) waves. At present, computations of poles and field, taking into consideration the transmission function, are not yet concluded. Therefore, there is no possibility of comparison with experimental results. Some preliminary evaluations, however, using this approach, have shown close agreement with experimental results.

Table 4

Transmission function of the atmosphere.

Frequency	Day			Night	
	$\exp\left\{-\frac{\omega}{c}\int xdz\right\}_-$	$\exp\left\{-\frac{\omega}{c}\int xdz\right\}_+$	$\left(\frac{v}{c}\right)_-$	$\exp\left\{-\frac{\omega}{c}\int xdz\right\}_-$	$\left(\frac{v}{c}\right)_-$
1	0,1	$9\cdot10^{-3}$	$1,8\cdot10^{-3}$	0,62	$2,8\cdot10^{-3}$
10	$3,8\cdot10^{-2}$	$1,5\cdot10^{-3}$	$2,1\cdot10^{-3}$	0,56	$3,6\cdot10^{-3}$
50	0,10	$\sim10^{-10^2}$	$3,10^{-3}$	0,72	$4,8\cdot10^{-3}$
10^2	0,18	$\sim10^{-10^4}$	$4,10^{-3}$	0,79	$5,3\cdot10^{-3}$
10^3	0,26	$\sim10^{-2,6\cdot10^4}$	$9,8\cdot10^{-3}$	0,83	$1,6\cdot10^{-2}$
$2\cdot10^3$	0,16	—	—	—	—
$5\cdot10^3$	$7,3\cdot10^{-2}$	—	—	—	—
$7\cdot10^3$	$8,3\cdot10^{-2}$	—	—	—	—
10^4	$2,9\cdot10^{-2}$	$\sim10^{-8\cdot10^3}$	$3,4\cdot10^{-2}$	0,59	$5,0\cdot10^{-2}$
10^5	$1,2\cdot10^{-5}$	$\sim10^{-2,3\cdot10^4}$	$1,0\cdot10^{-1}$	0,18	0,14
10^6	10^{-10^5}	$\sim10^{-5\cdot10^4}$	0,17	$\sim10^{-9\cdot10^4}$	0,53

It should be noted here that at frequency, $f < 10^4$ c/s, the average phase velocity is smaller than velocity, c, by two magnitudes and more, a fact which is linked with the magnetohydrodynamic character of waves at these frequencies.

Final Notes

As a matter of interest to readers of this translation, a review of an earlier paper of Al'pert is included here. This was reprinted from Math. Revs., vol. 21, No. 10, 1285 (Nov. 1960). Also, I am attaching copies of letters written to Borodina and Al'pert. These have never been answered.

Al'pert, Ya. L. Computation of the field of LF and VLF radiowaves over the Earth's surface under natural conditions. Radiotehn. i Elektron. **1** (1956), 281–292; available in translation **T320** by E. R. Hope. Directorate of Scientific Information Service, Defense Research Board, Ottawa, Canada, Nov. 1959.

The paper sets out to compute the time-harmonic electromagnetic fields in the frequency range 500 c/s to 30 kc/s at large distances along the surface of the earth, from a transmitting antenna. The adapted model is a parallel-plate waveguide whose lower boundary is the air-ground interface and whose upper boundary, at height h, is the lower edge of the ionosphere D region. The source is represented by a vertical electric dipole. The calculations are based on the general integral contour representation for the planar problem. This is transformed to a sum of modes which individually correspond to the residues of the poles of the integrand. The resulting pole-determining equation involves the reflection coefficient $\rho_2(C)$ of the upper boundary for plane waves incident at a (complex) angle whose cosine is C. The conductivity of the D region is allowed to vary with height $h+z$ according to the function $[1+\exp(-mz)]^{-1}$ where m is chosen to be 0.413 to correspond to best available published data (the influence of the earth's magnetic field is neglected). Taking P. S. Epstein's formula [Proc. Nat. Acad. Sci. U.S.A. **16** (1930), 627–637] for the reflection coefficient $\rho(C)$, the author then uses a semi-empirical technique described as the "alignment method", to estimate $\rho_2(C)$. This is the crucial step in the analysis and its implications are not adequately discussed in the paper. Actually, Epstein's expression for $\rho(C)$ is valid only for horizontal polarization (i.e., the electric vector is horizontal), whereas the required expression $\rho_2(C)$ for the pole equation corresponds to vertical polarization (i.e., the magnetic vector is horizontal). The "alignment" procedure amounts to using Epstein's results for horizontal polarization to define an equivalent sharply bounded ionosphere whose conductivity is, strictly speaking, both a function of C and the frequency. It is then assumed that this same "effective" conductivity may be inserted into the usual Fresnel reflection formula for vertical polarization to yield a numerical value for $\rho_2(C)$. In view of the radically different behavior of the Fresnel reflection coefficients for horizontal and vertical polarization at grazing angles (i.e., small C) the validity of this procedure is doubtful.

The author then claims that his results may be corrected for the sphericity of the earth by multiplying the fields by the factor $(\theta/\sin\theta)^{1/2}$ where θ is the central angle. The adequacy of this step is questioned since the earth

curvature modifies the pole-determining equation and the attenuation of the dominant modes would be increased if this were taken into account. This fact would have been evident if the problem had been formulated in spherical geometry at the outset. Actually, an earth-flattening approximation for the pole-determining equation is only valid when the spherical wave functions are adequately represented by their second order or Debye approximation. Such is the case when $(ka/2)^{1/3}C > 1$, where k is the wave number and a is the radius of the earth. This appears to be violated for many of the numerical results given in the paper. In fact, if curvature is completely neglected, a further restriction (which could be obtained from geometrical optics) is that $C > (h/a)^{1/2} \approx 0.1$. This is also violated in many of the cases treated by the author.

It is possible that the reported agreement with experimental data, in the frequency range 20 to 30 kc/s, is partly fortuitous since the "alignment procedure", mentioned above, yields overly high attenuation of the nodes, whereas the neglect of earth curvature leads to very ow attenuation. *J. R. Wait* (Boulder, Colo.)

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
BOULDER LABORATORIES
BOULDER. COLORADO
March 3, 1961

CRYOGENIC ENGINEERING LABORATORY
CENTRAL RADIO PROPAGATION LABORATORY
RADIO STANDARDS LABORATORY

IN REPLY
REFER TO FILE NO.

79. 70

AIR MAIL

＊ Dr. S. V. Borodina
Institute of Terrestrial Magnetism, Ionosphere and Propagation
Moskovskaya obl. , Leninskiu r-n
P/O Vatutenki, USSR

Dear Dr. Borodina:

I have been very interested in your paper "Review of the Modern State on the Propagation of Ultralong Electromagnetic Waves, " co-authored by Iu. K. Kalinin, G. A. Mikhailova, and D. S. Fligel. I have also been very interested in the papers of your distinguished colleague, Ya. L. Al'pert. Our laboratory would be very pleased to receive reprints of any of this work. We would also like a copy of your paper published in the works of NIZMIR 13, 3 (1959).

In exchange for the above material, I am sending to you under separate cover twenty reprints.

I was rather surprised about your comments on mode numbering. Isn't it really a matter of nomenclature? It is immaterial whether the dominant mode is labelled zero or one, provided that the excitation is properly accounted for. Similar remarks apply to the higher mode. Some further discussion on this point by Dr. H. H. Howe is contained in Vol. IV of the Proceedings of the 1957 Symposium on VLF radio waves. A copy of this is also being sent under separate cover.

I believe the question of sphericity of the earth is a very important factor. My view on this is expounded in some of the reprints being sent to you. In this connection I think you might also be interested in my review of Al'pert's article. A copy of this, taken from **Mathematical Reviews,** is enclosed. I would appreciate it if you could bring this to his attention. (I wasn't sure of his current address.) As you see, I am particularly concerned about the applicability of the flat-earth model to frequencies above 15 kc/s since the condition $(ka/2)^{1/3}$ C > > 1 is seriously violated. I would be very interested to know if you agree with my view on the subject.

Yours truly,

Encl
  Rev of Al'pert's
  article

James R. Wait, Consultant
Boulder Laboratories

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
BOULDER LABORATORIES
BOULDER, COLORADO

June 15, 1961

IN REPLY
REFER TO FILE NO.

79. 70

AIR MAIL

*

Dr. S. V. Borodina
Institute of Terrestrial Magnetism,
 Ionosphere and Propagation
Moskovskaya obl.
Leninskiy r-n
P/O Vatutenki
USSR

Dear Dr. Borodina:

I was wondering if you received my earlier letter (dated March 3, 1961) and a number of reprints. A copy of the letter is enclosed. I was hoping to receive some comments from you and reprints of some of your papers.

I have looked again into the question of the influence of ground conductivity on the VLF modes. I have been able to confirm our earlier calculations which were quoted in Fig. 4 of your paper in Izvestia V. U. Z., Vol. 3, pg. 12, Jan./Feb. 1960. This was done by carrying out a simple perturbation calculation using a method described in my paper in Proceedings of the Inst. of Radio Engineers, vol. 45, pg. 763, June 1957. This shows that for low-order modes the additional attenuation resulting from finite ground conductivity is given by

$$\frac{6.14}{h} (G)^{\frac{1}{2}} \times 10^3$$

decibels per 1000 km of path length, where $G = (\epsilon_0 \, \omega/\sigma)^{\frac{1}{2}}$. For example, if h = 80 km, f = 16 kc/s, $\sigma = 2 \times 10^{-3}$ mhos/m, we find that the additional attenuation due to ground losses is 1.6 db./1000 km. This agrees very closely with the difference between the two curves for $\sigma = 2.08$ and $\sigma = \infty$ in Fig. 5 of my paper on pg. 770 of the same issue of the journal. These latter results were obtained from a full numerical treatment of the exact mode equation on a digital computer.

Dr. S. V. Borodina   (2)   June 15, 1961

  Could there be an error of amount $\sqrt{2}$ in your equation (10)?
This might resolve the discrepancy. It is also possible that in
transferring my results in MKS units to your CGSE system an error
crept in.

  I hope to hear from you shortly.

         Yours truly,

Encl   James R. Wait
cy ltr 3/3/61   Consultant
                                      Central Radio Propagation Laboratory

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
BOULDER LABORATORIES
BOULDER. COLORADO
September 25, 1961

CRYOGENIC ENGINEERING LABORATORY
CENTRAL RADIO PROPAGATION LABORATORY
RADIO STANDARDS LABORATORY

IN REPLY
REFER TO FILE NO.

AIR MAIL

79. 70

*
Dr. Ya. L. Al'pert
Institute of Terrestrial Magnetism, Ionosphere
 and Propagation
Moskovskaya obl.
Leninskiy r-n
P/O Vatutenki
U.S.S.R.

Dear Dr. Al'pert:

I have been reading some of the recent Russian literature on VLF propagation. In two places[†] you and your colleagues have used equation (25) in Bremmer's NBS Report 5518 (later published in NBS Jour. Res. Sec. D., July 1959). From this an earth-curvature correction to the flat-earth mode equation has been obtained. In the conventional notation this equation would read

$$i \frac{k_o^2}{k_1^2} \sqrt{k_1^2 - k_o^2} = \left[i k_o C_n + (\frac{S_n}{C_n})^2 \frac{1}{2a} \right] \times$$

$$\frac{[1 - R_i(C_n) \exp(i 2k_o h C_n)][1 + i(S_n/C_n)^2(2k_o a C_n)^{-1}]}{[1 + R_i(C_n) \exp(i 2k_o h C_n)][1 - i(S_n/C_n)^2(2k_o a C_n)^{-1}]}$$

where k_o and k_1 are the (complex) wave numbers for the air and earth, respectively, a is the radius of the earth, h is the height of the ionospheric reflecting layer whose reflection coefficient is $R_i(C_n)$, C_n and S_n are the usual complex and cosine and sines characterizing the mode. Now, I am wondering if this equation is valid since the difference between τ and τ' has been neglected. Is this not the same

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† A. L. Al'pert, Rastrostvanenie radiovoln i ionosfera, publ. by Acad. of Sci. USSR, 1960 and S. V. Borodina, Iu. K. Kalinin, G. A. Mikhailova, and D. S. Fligel, Izvestia, V. V. Z., Vol. 3, No. 1, pp. 5-32, Jan/Feb. 1960.

Dr. Ya. L. Al'pert (2) September 25, 1961

as saying that C_n and C'_n are the same? Actually

$$S'_n \cong \frac{a}{a+h} S_n$$

and

$$C'_n = [1 - (S'_n)^2]^{\frac{1}{2}} \cong [C_n^2 + \frac{2h}{a}]^{\frac{1}{2}}.$$

Consequently, should not $2h/a$ be much less than C_n^2 to allow τ to be replaced by τ'? This is a fairly stringent condition. Another criticism of this equation is that the indicated curvature correction can only be significant if $C_n^3 k_o a$ is not large compared with unity. But, the condition for the use of the asymptotic forms (eqns. 23 and 24) in Dr. Bremmer's paper is only valid if $C_n^3 k_o a$ is large compared with unity. Therefore, I question whether the results are valid under any practical condition. Borodina, et al. have taken this equation to obtain curvature corrections to the attenuation rates. In some cases their corrected value is ten times larger than the case for $a = \infty$. Obviously, this results from the smallness of $C_n^3 k_o a$.

I would be interested in your reply to my questions. Possibly, there is something which I have overlooked.

Reprints of a few recent papers of our (e.g., Wait in NBS Jour. Res. Sec. D, Mar./Apr. 1960 and Jan./Feb. 1961, also Wait and Spies in J. G. R., Aug. 1960 and March 1960), are enclosed. In return, I would appreciate reprints of any of your papers.

Yours truly,

Encls   James R. Wait
 Reprints 143, 145, 161, 169   Consultant
 Cy Wait review of   Central Radio Propagation Laboratory
 Al'pert paper in Math
 Revs, Nov. 1960
 Cy ltr dtd 6-15-61 to
 Dr. S. V. Borodina

P. S.   Incidentally, I have written two letters to Dr. Borodina, but as yet no reply has been forthcoming. A copy of the last one is enclosed. Possibly you could forward this to her.